AD-784 944

AGARD FLIGHT TEST INSTRUMENTATION SERIES. VOLUME 6. OPEN AND CLOSED LOOP ACCELEROMETERS

I. McLaren

Advisory Group for Aerospace Research and Development

Prepared for:

Royal Aircraft Establishment

July 1974

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

AGARD-AG-160
Volume 6

NORTH ATLANTIC TREATY ORGANIZATION

ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT

(ORGANISATION DU TRAITE DE L'ATLANTIQUE NORD)

AGARDograph No.160 Vol.6

OPEN AND CLOSED LOOP ACCELEROMETERS

by

I.McLaren

Volume 6

of the

AGARD FLIGHT TEST INSTRUMENTATION SERIES

Edited by

W.D.Mace and A.Pool

This AGARDograph has been sponsored by the Flight Mechanics Panel of AGARD.

46

THE MISSION OF AGARD

The mission of AGARD is to bring together the leading personalities of the NATO nations in the fields of science and technology relating to aerospace for the following purposes:

- Exchanging of scientific and technical information;
- Continuously stimulating advances in the aerospace sciences relevant to strengthening the common defence posture;
- Improving the co-operation among member nations in aerospace research and development;
- Providing scientific and technical advice and assistance to the North Atlantic Military Committee in the field of aerospace research and development;
- Rendering scientific and technical assistance, as requested, to other NATO bodies and to member nations in connection with research and development problems in the aerospace field;
- Providing assistance to member nations for the purpose of increasing their scientific and technical potential;
- Recommending effective ways for the member nations to use their research and development capabilities for the common benefit of the NATO community.

The highest authority within AGARD is the National Delegates Board consisting of officially appointed senior representatives from each member nation. The mission of AGARD is carried out through the Panels which are composed of experts appointed by the National Delegates, the Consultant and Exchange Program and the Aerospace Applications Studies Program. The results of AGARD work are reported to the member nations and the NATO Authorities through the AGARD series of publications of which this is one.

Participation in AGARD activities is by invitation only and is normally limited to citizens of the NATO nations.

The content of this publication has been reproduced
directly from material supplied by AGARD or the author.

Published July 1974

531.768.629.73.05:533.6.054

Printed by Technical Editing and Reproduction Ltd
Harford House, 7–9 Charlotte St, London, W1P 1HD

PREFACE

Soon after its foundation in 1952, the Advisory Group for Aeronautical Research and Development recognized the need for a comprehensive publication on flight test techniques and the associated instrumentation. Under the direction of the AGARD Flight Test Panel (now the Flight Mechanics Panel), a Flight Test Manual was published in the years 1954 to 1956. The Manual was divided into four volumes: I. Performance, II. Stability and Control, III. Instrumentation Catalog, and IV. Instrumentation Systems.

Since then flight test instrumentation has developed rapidly in a broad field of sophisticated techniques. In view of this development the Flight Test Instrumentation Committee of the Flight Mechanics Panel was asked in 1968 to update Volumes III and IV of the Flight Test Manual. Upon the advice of the Committee, the Panel decided that Volume III would not be continued and that Volume IV would be replaced by a series of separately published monographs on selected subjects of flight test instrumentation: the AGARD Flight Test Instrumentation Series. The first volume of this Series gives a general introduction to the basic principles of flight test instrumentation engineering and is composed from contributions by several specialized authors. Each of the other volumes provides a more detailed treatise by a specialist on a selected instrumentation subject. Mr W.D.Mace and Mr A.Pool were willing to accept the responsibility of editing the Series, and Prof. D.Bosman assisted them in editing the introductory volume. AGARD was fortunate in finding competent editors and authors willing to contribute their knowledge and to spend considerable time in the preparation of this Series.

It is hoped that this Series will satisfy the existing need for specialized documentation in the field of flight test instrumentation and as such may promote a better understanding between the flight test engineer and the instrumentation and data processing specialists. Such understanding is essential for the efficient design and execution of flight test programs.

The efforts of the Flight Test Instrumentation Committee members and the assistance of the Flight Mechanics Panel in the preparation of this Series are greatly appreciated.

T.VAN OOSTEROM
Member of the Flight Mechanics Panel
Chairman of the Flight Test
Instrumentation Committee

CONTENTS

LIST OF SYMBOLS

Symbol	Meaning
A	amplitude
	amplitude of input forcing function
	acceleration
A_C	acceleration normal to sensitive axis
A_N	acceleration along sensitive axis
a	acceleration
b	damping constant
b_c	critical damping constant
c	arbitrary constant
D	drag
d	displacement
	arbitrary constant
e	base of natural logarithms
F	force
f	frequency in Hertz
f_n	undamped natural frequency in Hertz
$f(t)$	general function of time
g	acceleration of gravity
H	magnetic field strength, height
I	moment of inertia
I_c	current
k	spring constant
	spring rate of all spring-like forces or torques
k_m	force or torque per unit current in force coils
k_G	total gain of amplifier, pick-off and feed-back circuit
L	lift force
ℓ	length
ℓ_A	distance between pendulum axis and axis of impressed angular acceleration
m	mass
N	ratio of impressed frequency to natural frequency
n	revolutions per second
P	period of oscillation
p	operator
r	radius
S	Laplace transform operator
s	second
T	torque, thrust
t	time
V	voltage, velocity
W	weight
X, Y, X_1, Z	displacement

LIST OF SYMBOLS (concluded)

Symbol	Meaning
α	
ϕ	angle
θ	
γ	
ζ	damping factor relative to critical damping
δ_c	cross-coupling coefficient, aqual to the angle moved through by the arm per g of acceleration along the sensitive axis
δ_{st}	static spring deflection
ω	angular velocity
	circular frequency in rad/s
ω_n	undamped natural frequency in rad/s

OPEN AND CLOSED LOOP ACCELEROMETERS

I. McLaren
Royal Aircraft Establishment,
Bedford, England

SUMMARY

The aim of this Paper is to provide a balanced up-to-date presentation of the state-of-the-art of both open and closed loop accelerometers used for aircraft flight test work, covering system and component analysis and basic hardware design.

Both physical design problems and mathematical analysis are covered and special emphasis is put on those aspects likely to be relevant to aircraft flight test work.

Performance details include repeatability, compensation for temperature variation, insusceptibility to cross effects, stability under vibration and frequency response.

The application of accelerometers in several flight test techniques together with their performance requirements are reviewed in order to guide the flight test engineer in making his choice of instrument in any particular case.

The principles of frequency response tests are discussed in association with the theoretical characteristics of various, nominally, second order systems which are modified either by the method of testing or by the inherent, practical difficulties of instrument design.

The subject is a wide one and the absence of the treatment of any specific instrument or technique is not to be interpreted as an indication that they are unimportant. Rather it is an admission that lack of space and time precluded their inclusion.

Fortunately, many excellent publications by manufacturers and other authors are available and a comprehensive list of references is included for readers who wish to study specific subjects from other sources.

Structural and flutter applications of accelerometers have not been covered in this Paper because these subjects are specialized and should rightly be subject to separate treatment.

1 INTRODUCTION

Accelerometers belong to that family of inertial sensors which make measurements in terms of Newton's laws. Unlike displacement and velocity, which are frequently determined with respect to arbitrary reference levels, acceleration can be measured on an absolute basis.

The title obviously covers a host of devices and for the purpose of this Paper the discussion has been confined to those instruments which are used to determine the linear and rotational accelerations of the whole or part of the rigid body of an aircraft by sensing inertia effects within the vehicle itself.

Accelerometers have been firmly established in the field of flight test work for several decades and they have become indispensable tools of the flight test engineer. They are used to determine the various linear and angular accelerations that the aircraft experiences from control inputs, gusts change of thrust or aerodynamic drag etc. It is probably true to say that few, if any, tests concerned with research into aircraft flight mechanics are conducted without the aid of these inertial instruments.

The ever-widening scope and ever-increasing calibre of flight test work demand new measuring techniques which, in turn, require new and improved methods of measurement. Consequently, a high order of technical judgement is required to decide which of the various techniques and transducers for making measurements are most appropriate for any given case.

The subject is dealt with from the viewpoint of a flight test instrumentation engineer whose concern is to guide the flight test engineer in selecting the correct transducer for a particular job. With this in view the author has aimed at familiarising the flight test engineer (present or aspiring) with as many aspects of the subject as possible to enable him to make a clear choice.

A large number of new and sophisticated transducers of the closed loop or force feedback type are now readily available and an attempt is made to present the theoretical and practical background necessary for an understanding of the behaviour of these instruments. Both physical design problems and mathematical analysis are covered and special emphasis is put on those aspects likely to be relevant to aircraft flight test work.

Naturally, in the circumstances, it is often easy to forget some of the simple conventional open loop types of transducer which held sway several years ago. Their performance, from considerations of expediency of completing a programme, dynamic performance, cost and reduction of complexity may be adequate for certain applications and indeed, may even be superior to that of the more elegant types. For these reasons a survey would not be complete without them.

The main purpose of the text is devoted to relating the actual problems associated with the use of particular transducers to the measurement techniques for flight test work. This requires that the basic fundamentals and general principles of each type must be reviewed and amplified. Although much of the mechanical physics involved in the operation of the instruments is well-known, they are included herein with no apology because they must be appreciated if practical solutions are to be sought.

2 OPEN LOOP LINEAR ACCELEROMETERS

2.1 Basic theory

Linear accelerometers determine the acceleration of an aircraft by measuring the so-called forces of inertia always appearing in non-uniform or curvilinear motion. The most commonly used arrangement for measuring linear acceleration is the damped mass-spring system. Fig.1 illustrates a simple idealized single axis accelerometer.

Its action is based on measurement of the movement of an elastically suspended seismic mass installed in a housing rigidly attached to the aircraft being investigated. Dampers are used to provide means of controlling the response of the instrument to dynamic inputs. These accelerometers have been known for a long time and are still widely used in flight test work.

The operation of an accelerometer is based on Newton's second law: a finite mass opposes acceleration with a force proportional to the product of the mass and the acceleration. Referring to Fig.1, the device consists of a seismic mass m, restrained by a spring of stiffness k and damped by a viscous friction force $b\ dX/dt$. The mass is constrained to deflect only along the measuring axis of the instrument and the damping force $b\ dX/dt$, is proportional to the relative velocity of the mass and opposes its motion. Displacement of the mass is sensed by a pick-off.

Let the frame be subjected to accelerated motion in inertial space along the measuring axis of the accelerometer. Assume that the frame at some time during the motion is at a distance Y from its rest position and at this time also assume that the mass has travelled a distance Z in inertial space from its rest position. Hence, the relative displacement of the mass to the frame is then $Z - Y$, which can be represented by X.

It is clearly seen that the equation of motion for the mass m acted on by the two forces - the restoring force exerted by the spring and equal to kX, and the damping force equal to $b\ dX/dt$ - is:

$$-kX - b\frac{dX}{dt} = m\frac{d^2Z}{dt^2} \tag{1}$$

but

$$Z = X + Y$$

therefore

$$-kX - b\frac{dX}{dt} = m\frac{d^2(X+Y)}{dt^2} \tag{2}$$

and

$$\frac{d^2X}{dt^2} + \frac{b}{m}\frac{dX}{dt} + \frac{k}{m}X = -\frac{d^2Y}{dt^2} \quad . \tag{3}$$

The two sides of Eq(3) must be of different sign because the deflection of the mass relative to the case is in the opposite direction to the applied acceleration. Putting $\omega_n = (k/m)^{\frac{1}{2}}$ = undamped natural circular frequency* and $\zeta = b/2m\omega_n$ = damping factor* relative to critical we have

$$\frac{d^2X}{dt^2} + 2\zeta\omega_n\frac{dX}{dt} + \omega_n^2X = -\frac{d^2Y}{dt^2} \quad . \tag{4}$$

This is the general equation of a second-order system relating displacement of the mass to any arbitrary input Y.

The steady state displacement of the mass m to a constant acceleration a is:

$$\omega_n^2X = \frac{d^2Y}{dt^2} = a \tag{5}$$

or

$$X = \frac{m}{k}a \quad . \tag{6}$$

2.2 Sinusoidal excitation

The frequency response function, i.e. response to sinusoidal excitation, is obtained by the Laplace transformation to provide both the amplitude and the phase angle of the response spectrum. If the driving function is simple harmonic motion, $Y = A \sin \omega t$, then from Eq(4):

$$\ddot{X} + 2\zeta\omega_n\dot{X} + \omega_n^2X = A\omega^2 \sin \omega t \quad . \tag{7}$$

Applying the Laplace transformation to both sides of this equation and expressing $\bar{X}(S)$ as the transform of the function $X(t)$ yields:

$$S^2\bar{X}(S) - SX(0) - \dot{X}(0) + 2\zeta\omega_n\{S\bar{X}(S) - X(0)\} + \omega_n^2\bar{X}(S) = \frac{A\omega^3}{(S^2 + \omega^2)} \tag{8}$$

where $\dot{X}(0)$ and $X(0)$ are the initial conditions of $X(t)$.

Noting that the initial velocity and displacement are both usually zero, and solving for $\bar{X}(S)$, Eq(8) becomes:

$$\bar{X}(S) = \frac{A\omega^3}{(S^2 + \omega^2)\left(S^2 + S2\zeta\omega_n + \omega_n^2\right)} \quad . \tag{9}$$

---

* ω_n and ζ are derived in Appendix 1 and 2.

By resolving Eq(9) into partial fractions with linear complex denominators and taking the inverse transform, the solution is expressed as:

$$X(t) = \frac{A\omega^2 \sin(\omega t - \phi)}{\omega_n^2\left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2\right]^{\frac{1}{2}}} + \begin{pmatrix}\text{transient term decaying}\\ \text{exponentially with time}\end{pmatrix} \qquad (10)$$

where

$$\tan\phi = \frac{\left(\frac{2\zeta\omega}{\omega_n}\right)}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]} \; . \qquad (11)$$

The first term of the right hand side of Eq(10) is the desired steady state solution (particular integral). Therefore the output-input relationship is:

$$\frac{\omega_n^2 X}{A\omega^2} = \frac{1}{\left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2\right]^{\frac{1}{2}}} \qquad (12)$$

and letting $N = \omega/\omega_n$ the amplitude of the response spectrum is given by:

$$X(\omega) = \text{mod } X(J\omega) = \left|[(1 - N^2)^2 + (2\zeta N)^2]^{-\frac{1}{2}}\right| \qquad (13)$$

and the phase angle by:

$$\tan\phi = \frac{2\zeta N}{1 - N^2} \; . \qquad (14)$$

Linear second order systems can be completely described by two fundamental parameters; the undamped natural frequency (f_n) and the damping factor relative to critical ζ. By referring to the well-known family of response curves Figs.2 and 3, the moduli and phase shifts of these systems can be determined for sinusoidal excitation at any frequency.

These response curves in Fig.2 can be represented by two straight lines to which the curves become asymptotic for values of N below 0.4 and above 2.5. It is seen that for frequencies below the natural frequency the relative amplitude of the mass is proportional to the applied acceleration, whilst for frequencies above the natural frequency the amplitude of the mass is proportional to the amplitude of motion of the vibrating body. In the latter case the mass of the accelerometer behaves like a free mass and registers displacement and this region represents that part of the frequency spectrum over which seismographic instruments such as vibrographs and displacement pick-ups are used.

Mass-spring systems can be used to measure velocity or displacement in addition to acceleration. In fact, this family of pick-ups respond as accelerometers in the region below their natural frequencies, as velocity meters near their natural frequencies, and as vibrometers or displacement meters in the region well above their natural frequencies. Since there are few velocity measuring units of this type now in use, further discussion of these items is not justified.

The equation for the response curve of a displacement pick-up unit is derived from Eq(12), and is given by

$$\frac{X}{A} = \frac{N^2}{[(1 - N^2)^2 + (2\zeta N)^2]^{\frac{1}{2}}} \; . \qquad (15)$$

In other words, the response is the ratio of the amplitude of vibration of the mass to the amplitude of the applied vibration. Response curves for displacement pick-up units for different values of damping are shown in Fig.4.

2.3 Transient response

The response of our idealised system to a transient forcing function is now considered. Frequently, the desired characteristics of instrument performance are readily interpreted in terms of the transient response.

From a knowledge of the manner in which a system approaches or returns to a steady state after the application of step or impulsive inputs respectively, the engineer can estimate the value of the damping factor and natural frequency of the system.

In general, acceleration steps of sufficiently fast rates of acceleration change are physically difficult to realise. However, in those cases where access can readily be gained to the mass-spring system a step-function input can be simulated by giving the mass an initial displacement and suddenly releasing it from this position. The transducer system can then be evaluated on the basis of the recorded response to a step-function input as if a pure step-function input of acceleration had been impressed on the instrument.

In many instances, however, access cannot be obtained to the moving parts and methods employing impulsive techniques must be resorted to. The impulsive function in dynamics can be considered as a very large force acting for a very short time. Perhaps the most simple method of generating an impulse function is to rest the accelerometer case on a table on one of its edges and snap down the case on to the table. When the case makes contact with the table kinetic energy is imparted to the transducer and this initiates movement of the mass-spring system relative to the case.

Factors which must be taken into account are the relation between the rate of application of the force and rate of deflection of the mass-spring system and also movement of the accelerometer case must have ceased immediately when the face of the case made contact with the table. Since the forcing function cannot be determined precisely the amplitude *versus* time plot of the transducer output is not evaluated until after the recorded response has reached its first peak value. By this time the mass-spring system should be executing free vibrations and the response of the system is evaluated on the basis of the response to a step-function input.

Although a linear system may be completely characterized by its response to any aperiodic signal it is usual to consider the step function. Since most second order systems of practical interest are under-critically damped (oscillatory), only this case will be considered. The method makes three basic assumptions:

(1) The system is linear, and therefore possesses a Laplace transform.

(2) The system is at rest before application of the transient forcing function.

(3) The forcing function is a unit step applied at time equal to zero.

The response to a unit step of acceleration, when ζ is less than 1, is obtained from the characteristic differential equation of motion for a single-degree-of-freedom transducer. This is written as:

$$m\ddot{X} + b\dot{X} + kX = 0 \quad . \tag{16}$$

This equation is acted on by a step function which is expressed as a function of time thus:

$$f(t) = 0 \text{ for } t < 0$$

$$f(t) = F \text{ for } 0 < t \leq \infty \quad .$$

Therefore the equation of motion of the system under analysis is:

$$m\ddot{X} + b\dot{X} + kX = f(t) \quad . \tag{17}$$

Applying the Laplace transform to this equation, and with initial values $\dot{X} = 0$, and $X = 0$ (even if the latter is not zero, it will have a constant displacement which can be eliminated for the purpose of analysis by a shift in the coordinate system), yields:

$$S^2 m\bar{X}(S) + Sb\bar{X}(S) + k\bar{X}(S) = \frac{F}{S} \quad . \tag{18}$$

Re-arranging and substituting $\omega_n = (k/m)^{\frac{1}{2}}$, and $b/m = 2\zeta\omega_n$, gives:

$$\bar{X}(S) = \frac{F}{m} \frac{1}{S\left(S^2 + 2\zeta\omega_n S + \omega_n^2\right)} \quad . \tag{19}$$

Letting

$$\omega_n(1 - \zeta^2)^{\frac{1}{2}} = c$$

and

$$\zeta\omega_n = d$$

and taking the inverse transform, yields the response function

$$X(t) = \frac{F}{m\omega_n^2}\left\{1 - e^{-dt}\cos ct - \frac{d}{c}e^{-dt}\sin ct\right\} \quad . \tag{20}$$

Reverting to original values for c and d and re-arranging gives:

$$X(t) = \frac{F}{m\omega_n^2}\left\{1 - \frac{e^{-\zeta\omega_n t}}{(1 - \zeta^2)^{\frac{1}{2}}}\left(\sin\left[(1 - \zeta^2)^{\frac{1}{2}}\,\omega_n t + \phi\right]\right)\right\} \tag{21}$$

where $\phi = \tan^{-1}(1 - \zeta^2)^{\frac{1}{2}}/\zeta$ and $F/m\omega_n^2$ is the steady-state displacement as $t \to \infty$.

In order to put Eq(21) in a more compact and workable form, let

$$X_1 = 1 - \frac{m\omega_n^2}{F} X$$

$$A = \frac{1}{(1 - \zeta^2)^{\frac{1}{2}}}$$

and

$$\omega = (1 - \zeta^2)^{\frac{1}{2}} \omega_n \quad .$$

Then Eq(21) can be written with these new parameters as

$$X_1 = Ae^{-\zeta\omega_n t} \sin(\omega t + \phi) \quad . \tag{22}$$

This is the equation of a damped sinusoid.

Now the difference between the maximum displacement of a system subjected to a step input and the ultimate displacement is defined as the overshoot. It is found as a percentage of the final displacement from setting the derivative of Eq(22) equal to zero, and solving for the $\underline{\omega t}$ product at the first maximum point. The per cent overshoot is a function of the damping factor only and is given by:

$$\text{per cent overshoot} = 100\, e^{\left(-\frac{\zeta\pi}{(1-\zeta^2)^{\frac{1}{2}}}\right)} \quad . \tag{23}$$

The responses of a second order system (with various damping factors) to a step input are illustrated in Fig.5 together with a tabulated list of per cent overshoots.

It can be shown that the transfer function of the system, in Laplace notation, relating displacement to acceleration is:

$$\frac{x}{a}(S) = \frac{\frac{m}{k}}{\left(\frac{m}{k}\right)S^2 + \frac{b}{k} S + 1} \quad . \tag{24}$$

This is often expressed in the form:

$$\frac{\omega_n^2 x}{a} (S) = \frac{1}{1 + \left(\frac{2\zeta}{\omega_n}\right)S + \left(\frac{1}{\omega_n^2}\right)S^2} \quad . \tag{25}$$

3 CLOSED LOOP ACCELEROMETERS

3.1 Basic principles

Closed loop accelerometers or, as they are commonly called, force feedback acceleration transducers have a precision which is much superior to that of open loop types. Their accuracy is such that the ratio of the maximum measured acceleration to the minimum is of the order of 5000 or more, and many proprietary transducers are obtainable with this kind of performance.

The achievement of this high accuracy is due to the use of the so-called electric spring. Creation of a mass-spring accelerometer with such a ratio of maximum to minimum measured accelerations is difficult because practical defects caused by the zone of insensitivity, hysteresis of elastic suspension and change of spring rate due to temperature changes, to mention just a few, all combine to limit the accuracy of measurement.

By making use of the simplified servoed accelerometer illustrated in Fig.6, it is possible to develop and present the basic principles of closed loop accelerometers in a simple, physical way.

Let $\underline{V}$ be the voltage produced at the pick-off due to movement of the mass relative to the frame, and let $\underline{I_c}$ be the current at the amplifier output feeding into the restoring coil situated in a magnetic field. If $\underline{V}$ is proportional to movement $\underline{X}$ of the mass $\underline{m}$ of the accelerometer, the current $\underline{I_c}$ at the amplifier output will also be proportional to movement $\underline{X}$.

This current grows in the restoring coil to counteract movement of the mass. Thus, for an accelerating force on the frame acting along the measuring axis, the inertia of the mass causes it to lag behind the movement of the frame until the product of mass and acceleration is equalled by the force from the electro-magnetic restoring coil.

Consequently, when the mass has reached its final displacement th electro-magnetic feedback force acting on the mass is balancing the impressed acceleration force and hence the current flowing in the restoring coil or the output voltage appearing across resistor $\underline{R}$ will be accurately proportional to the impressed acceleration.

It follows that under equilibrium conditions, the following simple relationship holds:

$$ma = k_m I_c \tag{26}$$

or

$$I_c = \frac{m}{k_m} a \tag{27}$$

where I_c = force coil current
m = mass of sensitive element
a = acceleration along sensitive axis
k_m = force to balance sensitive element per unit force coil current.
(k_m depends on coil dimensions, coil current, and the strength of the magnetic field H.)

It is evident from inspection of Eq(27) that, provided the amplifier is not driven into saturation and that the working range of the pick-off is not exceeded, the steady state accuracy is:

(a) mainly dependent on the constancy of the magnetic field $\underline{H}$ in the air gap in which the force coil works

(b) independent of amplifier characteristics, and

(c) unlike that of a simple mass-spring system, independent of the displacement of the mass.

In fact, the pick-off, amplifier and restoring coil working in its magnetic field can be considered as constituting the components of a negative feedback loop which behaves in a manner similar to that of a mechanical spring but without having the latter's unsatisfactory characteristics.

It is also known that

$$I_c \propto k_G X \tag{28}$$

where k_G = total gain of amplifier, pick-off, and feedback circuit,
and X = displacement of sensing element.

Therefore, for a given acceleration, i.e. given current, change of system gain k_G alters the working displacement of the mass without affecting the steady state sensitivity. Hence the effects of changes in amplifier characteristics are eliminated and basically there is no limit to the smallest acceleration that can be detected and measured.

Referring to the feature listed under (a) it is clearly seen why high accuracies are obtainable from these instruments when it is considered that changes of magnetic stability in properly stabilised and protected magnets rarely exceed 5 parts in 100 000.

The preceding simplified analysis of the behaviour of closed loop accelerometers is sufficient to show their more important properties under steady state conditions. For instance, it is now evident that the most important advantage emerging from the discussion is the fact that servoed accelerometers make use of the principle that acceleration is measured directly by the value of current (or voltage) which achieves force balance rather than from the indirect measurement of displacement of the mass-spring system as occurs in the traditional accelerometer.

3.2 Dynamic characteristics

Our elementary force feedback accelerometer is also useful in exploring the dynamic characteristics of this type of instrument. By translating the physical description of the system characteristics in Fig.6 into mathematical terms the system equation is determined.

It has been shown that the mass $\underline{m}$ is acted on by a feedback force such that

$$\text{feedback force} = -k_m I_c \ . \tag{29}$$

Also, as $I_c = k_G X$, we can write, for a given gain k_G, that

$$\text{feedback force} = -kX \tag{30}$$

where k = force per unit displacement.

The mass is constrained to deflect only along the measuring axis of the instrument and the damping force $\underline{b\ dX/dt}$ is proportional to the relative velocity of the mass and opposes its motion. Under these conditions, if the accelerometer is subjected to any arbitrary acceleration d^2Y/dt^2 the following system equation can be written:

$$m \frac{d^2}{dt^2}(Y + X) = -b\frac{dX}{dt} - kX \tag{31}$$

or

$$\frac{d^2X}{dt^2} + \frac{b}{m}\frac{dX}{dt} + \frac{kX}{m} = -\frac{d^2}{dt^2}Y \ . \tag{32}$$

Comparing the system equations for the open and closed loop types, it is clearly seen that our two representative accelerometers are dynamically similar and are, in fact, simple second order systems. At first glance it would also appear that servoed accelerometers offer only an increase of accuracy over open loop types. However, this is not so and there are, of course, other advantages which will now be described.

It can be shown that the undamped natural frequency of simple mass-spring systems is:

$$f_n = \frac{1}{2\pi}\left(\frac{g}{d}\right)^{\frac{1}{2}} \tag{33}$$

where g = acceleration due to the earth's gravitational field, and
d = deflection of mass for an acceleration of 1 g.

In other words, the natural frequency is related to the deflection per g. This is a fundamental feature of these systems.

Now for a given acceleration the current I_c and hence the product $k_G X$ is a constant. It follows that the natural frequency or stiffness of the system can be altered by changing the gain k_G without altering the steady state sensitivity. For instance, increasing the gain by a factor of four doubles the frequency.

Moreover, in practice the servo controller usually includes differentiating and integrating networks to obtain the required dynamic response characteristics and for stability purposes. The addition of these networks means that feedback damping instead of mechanical damping, i.e. oil and eddy current damping, which are sometimes difficult to apply, can now be incorporated in the system.

Thus means are to hand to change both the frequency and damping to enable tailored frequency response characteristics to be produced. There would be considerable difficulty in providing this degree of flexibility in open loop types.

Unfortunately, many servo devices incorporating active networks in their closed loops constitute third order systems. For these systems the input-output phase angle may vary non-linearly with frequency in order that stability criteria be satisfied. In general, this variation will cause distortion, time lags of different magnitudes being introduced for different frequency components in the measured acceleration.

4 FUNDAMENTAL BEHAVIOUR OF AN ACCELEROMETER

The foregoing has served to explain the basic principles of simple mass-spring and servoed accelerometers. However, the development of the system's differential equation (4) tends to imply that the devices only detect forces which cause a movement of the main body in which they are housed. In fact, this movement can be influenced by accelerations due to gravity in addition to those caused by forces of non-gravitational origin, for example, engine thrust, lift and drag. The accelerometer does not measure any component of the acceleration due to the force of gravity unless an equal and opposing force is exerted on its body. This is due to the fact that the gravitational field influences simultaneously both the aircraft and the seismic mass of the accelerometer and so by itself produces no deflection of the mass-spring system. An accelerometer cannot distinguish between kinematic accelerations and mass attraction and hence the effects of any component of gravity acting along the accelerometer input axis must be allowed for if it is required to deduce the flight path of the aircraft. Where aerodynamic forces or structural loads are required no such allowance will be necessary.

What happens in practice can conveniently be visualized by considering in more detail just how our simple mass-spring accelerometer works. For instance, consider an accelerometer positioned on a flat platform such that its measuring axis is parallel to the platform surface.

If the platform is held horizontally and at rest, the system will be in equilibrium with zero indication of the accelerometer. Now, if the platform is tilted over an angle θ while the accelerometer is held on, it will indicate the component of gravity $g \sin\theta$. If the accelerometer is allowed to slide, in the absence of frictional forces, it will experience an acceleration of $g \sin\theta$ along the tilted platform, and indicate zero. With friction the indication will be proportional to the friction force only.

When the instrument is moved along the platform with a given non-gravitational acceleration, the inertia of the mass causes it to lag behind the movement of the case until the spring is stretched to the point that it exerts on the mass a force equal to the product of the mass and acceleration. This deflection of the mass from its rest position can be read as an indication of this non-gravitational force exerted on the instrument.

Now, if the accelerometer is placed on the platform so that its measuring axis is vertical, mass attraction is acting along the sensitive axis of the accelerometer and pulling the seismic mass toward the platform until it is balanced by spring tension. Observing and interpreting this output just as in the horizontal case, it might be inferred that the instrument was being subjected to vertical motion and that, in fact, it was being accelerated upward at 1 g. This misinterpretation is, as previously stated, due solely to the fact that the accelerometer cannot distinguish between path accelerations and mass attraction. From a physical point of view and endeavouring to present deductive reasoning in words rather than in symbolic form, the platform can be considered to push up on the instrument with a measurable force which would produce an actual upward acceleration of 1 g if gravity itself were not also operating.

Similarly, an aircraft in steady, level flight will require an upward force mainly from the wing lift, exactly counteracting the acceleration of gravity, while thrust and drag must be balanced in the horizontal plane. Earth-oriented accelerometers - e.g. mounted on an earth-oriented stabilized platform - would in this case indicate A_v = 1 g in the true vertical and A_h = 0 g horizontally.

In most practical cases the accelerometers will be fixed to the aircraft, generally oriented in the direction of the aircraft axes X , Y and Z . In the above case (level flight) the X- and Z-axes will be tilted relative to earth-oriented axes over an amount equal to the angle of attack α . As the acceleration resulting from vectorial addition of its three components should still be 1 g the following simple relations exist for the case of steady, straight, level flight (with wing level):

$$A_z = g\cos\alpha\ , \quad A_x = g\sin\alpha\ , \quad \text{and} \quad A_y = 0\ .$$

The kinematic acceleration of the aircraft follows from vectorially subtracting the acceleration of gravity from the resultant measured acceleration. In the above case the kinematic acceleration will then be found to be zero, indicating steady flight, as was the initial assumption. If the wing lift is made zero, the indication of the vertical accelerometer will drop to zero, in this case the aircraft follows a ballistic trajectory. As was mentioned before, rather than gravity the on-board accelerometers, according to their orientation, indicate the external non-gravitational forces that work on the aircraft, i.e. lift, drag, and thrust. They do so irrespective of the attitude of the aircraft.

In a steady climb (see upper part of Fig.7), again the resultant acceleration A_T is true vertical and equal to 1 g. In this case, however, the X- and Z-axes are tilted over an angle $\theta = \alpha + \gamma$, making the accelerometer indications:

$$A_z = g\cos\theta = g\cos(\alpha+\gamma) \quad \text{and} \quad A_x = g\sin\theta = g\sin(\alpha+\gamma)\ . \tag{34}$$

If the angle of attack α is known the climb angle γ - and therefore the aircraft's excess performance or excess power - can be deduced from the accelerometer readings with the above equations. The same holds true in the general case where kinematic acceleration is not zero and the resultant accelerometer indication is no longer oriented vertically (see lower part of Fig.7). In this case an equivalent climb performance can be determined which is the sum total of the aircraft's actual climb performance during the test and its residual acceleration performance. As it is total excess performance which is the desired quantity in performance flight testing, it is not necessary to determine climb and acceleration separately.

In general, lift and excess thrust $\Delta T = T - D$ (respectively normal to and in the flight path) can be determined from accelerometer indications by transferring the accelerometer measurements from aircraft axes to wind axes as follows (see upper part of Fig.7):

$$\left.\begin{aligned} A_n &= A_x\sin\alpha + A_z\cos\alpha = gL/W \\ A_p &= A_x\cos\alpha - A_z\sin\alpha = g(T-D)/W = dV/dt + g\sin\gamma \end{aligned}\right\}\ . \tag{35}$$

Aircraft performance or Specific Excess Power (SEP) can then be determined thus:

$$SEP = V\Delta T/W = (V/g)dV/dt + V\sin\gamma = (V/g)dV/dt + dH/dt = \frac{d/dt}{mg}(\tfrac{1}{2}mV^2 + mgH)\ .$$

This determination of climb or acceleration performance from accelerometer indications only requires knowledge of the angle of attack α , e.g. from vane indication. In general, A_y can be deleted as this determines the aerodynamic force due to sideslip, which is generally zero. Some misalignment of A_x and A_z relative to aircraft axes can be tolerated as long as α is calibrated (in steady, level flight) relative to the orientation of the longitudinal accelerometer, which then should remain invariant over the rest of the tests.

For simple tests like determination of stick force per g , generally the contributions of A_x and α are neglected and only A_z is used.

For VTOL aircraft α is meaningless in hovering flight. Therefore, if accelerometers are used they must be earth-oriented through using a stabilized platform. The flight path in this case is calculated from integration of the horizontal and vertical acceleration. A slight misalignment of the platform then will introduce errors as the resulting component of gravity will be interpreted as a horizontal acceleration.

A further application of accelerometers on a stabilized, earth-oriented platform is in inertial navigation, where extreme accuracy is required both in orientation and in indication of the accelerometer, as the position of the aircraft is calculated from double integration over a long period of time.

Since the quantity to be measured in many applications is the acceleration of the centre of gravity of the aircraft this should be the point of attachment of the instrument. However, the physical layout of many aircraft precludes the possibility of locating the instruments at this point. The fact that the instrument may be located at some distance from the centre of gravity involves the introduction of appreciable errors into the basic measurement. Interfering terms such as angular velocities and angular accelerations have now got to be considered and the analysis of the instrument's output has now to be carried out by writing the absolute acceleration of the instrument in terms of the aircraft centre of gravity motions. A comprehensive analysis of these interference terms and the evaluation of errors to be expected in particular installations are outlined in Vol.II, Chapter 11, of the AGARD Flight Test Manual[13]. Aero-elasticity also creates special problems because of the difficulties encountered in defining the orientation of the axis of the centre of gravity of a non-rigid aircraft.

5 ACCELEROMETER REQUIREMENTS FOR FLIGHT TEST WORK

It is convenient at this stage to give a short presentation of some of the flight test methods together with specific test objectives in which accelerometers are required for measuring purposes. This approach acquaints the reader with important challenging applications so that he is fully equipped to assess and appreciate the ability or otherwise of various types of transducers to satisfy numerous stringent requirements.

5.1 Dynamic stability tests

A typical dynamic investigation is the evaluation of the main aerodynamic stability derivatives, both lateral and longitudinal. One form of this type of test makes use of the free oscillation method which requires a control input to the aircraft to excite it in a known mode of oscillation.

In the particular study of Dutch roll characteristics physical parameters such as lateral acceleration, roll rate, and yaw rate amongst others are measured and recorded during the decay of the free oscillation. Aerodynamic derivatives in yaw, roll and sideslip are subsequently reduced from the recorded data.

The flight testing procedure requires the study of these properties at various values of lift coefficient but at reasonably constant Mach number and height. To achieve high values of lift coefficient it may be necessary to measure the aircraft's response to control pulses during turning manoeuvres involving high values of normal acceleration but during which Mach number and height vary slowly enough to leave the aerodynamic forces and moments relatively unaffected by the rate of change of these parameters. This means that the lateral accelerometer is measuring magnitudes of the order of 0.2 g or less in the presence of a 4g cross-axis component.

Furthermore, the semi-graphical nature of the time vector analysis technique[12] demands uncertainties of less than 1% in modulus and 1° or better in phase over a frequency range of 0-1 Hz from the measurement of the physical parameters to guard against gross mistakes. For instance, it has been found that an instrument error of 1° in phase angle measurement may represent an error of 10 to 15% in the modulus of the time vector representing the required force or moment which closes the vector polygon. These stringent requirements for transducers used for derivative extraction hold irrespective of the analysis technique used, e.g. response-curve fitting, Shinbrot etc. For tests where phase angle measurements are of special importance accelerometers with damping factors of about 0.75 critical damping are advantageous because the relative delay time, i.e. the phase shift time, which determines the distortion of the indication in multiples of the time of oscillation of the undamped system, is $0.25 f^{-1}$. In fact, with this damping factor, the phase lag is virtually proportional to frequency over a wide frequency band up to ω_n and at the same time the response modulus is very nearly unity up to say $N = 0.5$.

5.2 VSTOL tests

Severe environmental problems on VSTOL and STOL aircraft call for quantitative information on the thrust delivered by the lift engine. In practice, this is obtained from the measurement of vertical acceleration. Accelerometers are required to detect accelerations in the region of 5×10^{-4} g at ambient temperatures, in some cases, of up to 50° to 60°C, and in the presence of severe vibrational noise.

5.3 Gust research

Programmes of work directed towards investigations of atmospheric turbulence call for the measurements of both the gust input and the aircraft response. Tests of this nature may require the measurement of acceleration to better than 10^{-3} g at frequencies as low as 0.01 Hz.

5.4 Drag measurements

One of the quantities which may be of direct importance in the determination of the drag of an aircraft is the measurement of acceleration along the flight path. To enable the drag coefficient to be determined with reasonable accuracy this measurement is required to better than 10^{-3} g, possibly in the presence of high 'g' cross-accelerations and vibration. This means that flight path axes and aircraft datums relative to the accelerometer axes must be known to at least 10^{-3} radians.

5.5 Inertia measurement in flight

When dynamic properties of aircraft are being studied it is important to have a precise knowledge of the moment of inertia of the aircraft about a specific axis. A novel method for obtaining the yaw inertia consists of jettisoning a wing-tip parachute thus causing a step input of yawing moment and by measuring the resultant angular acceleration in yaw the yaw inertia is obtained. It has been found that for these tests an angular accelerometer is required to measure one or two degrees/s^2 to better than 1% and also it should have dynamic characteristics which are linear over a frequency range of 0-40 Hz. The latter feature enables transient inputs to be simulated on the instrument corresponding to those encountered in the tests so that calibration data is obtained consistent with the above accuracy. Although a pair of linear lateral accelerometers mounted at the nose and tail of an aircraft have been used for measuring yaw acceleration with satisfactory results, difficulties may be experienced with the use of this technique on less rigid aircraft.

6 PROPRIETARY TRANSDUCERS

6.1 Mass-spring accelerometers

Many types of open loop transducers are available from commercial sources and it is not the intention, within this Paper, to produce a documented survey or catalogue of these devices. Excellent market surveys on accelerometers have been reported in several periodicals and these include a comprehensive list of tabulated variables.

However, it is appropriate at this stage to discuss many of the basic principles and elements employed in these units to see how such features as steady state and dynamic accuracy, range, cross-effects and performance under environmental conditions are affected. Fortunately, a discussion of this kind can be kept within bounds when it is realized that the various kinds of accelerometers have a good deal in common, at least so far as concerns the critical mechanical and electrical elements. The damped mass-spring accelerometer is taken as a type instrument, many of the problems of which are pertinent to other kinds of accelerometers.

For several decades the damped mass-spring accelerometer has been the traditional tool used to measure the acceleration of an aircraft without reference to the external environment. Even today, when other types are available many measurements are still made with these units. In this type of accelerometer the force of a spring balances the acceleration force imposed on a mass. Typically, the acceleration is detected by measuring the deflection of the mass-spring assembly with a pick-off. Damping of the system is provided by dash-pots or, preferably, by a conducting ring or disc moving in a strong magnetic field (eddy current damping).

6.2 Spring suspensions

The sensing element is most frequently a mass suspended on springs. Spring configurations come in many forms the more common being cantilever and crossed leaf, spring hinge, pivoted arm with helical springs, and slotted diaphragm or so-called spider suspensions.

The proper design of these spring systems is of the utmost importance in the design of a satisfactory accelerometer. With improper design the best of component parts may give poor performance and the system as a whole may be subject to errors from several sources. For instance, by choosing first class spring material unwanted small displacements of the sensing element, resulting from temperature dependent dimensional changes and from hysteresis effects, are kept to a minimum. In addition, springs should have a linear load-deflection characteristic over the entire working range and a relatively high stiffness in all directions other than that in the measuring direction of the instrument.

It is also desirable to ensure that the mass moves in rectilinear motion. Spring assemblies, in which the sensing element rotates about some form of pivot or spring hinge, are the origin of some of the errors such as cross-coupling and rectification (refer to section 11.4) present in accelerometers. For instance, if the mass moves in an arc in the presence of cross accelerations, an output error which is related to the product of the cross and active acceleration components, is introduced.

6.3 Electrical pick-off

The electrical pick-off is employed to transform a mechanical input, such as a displacement, into an electrical output. Most present day accelerometers employ contactless pick-offs for this purpose. However, moving contact variable resistance devices (potentiometers) were used extensively some years ago and some are even in use today. They have inherent disadvantages and the criticisms normally levelled at these pick-offs are:

(a) mechanical wear may lead to erratic behaviour and eventually to failure,

(b) high operating force or torque is required to minimise friction, and

(c) the resolution is limited by the number of turns on the slide-wire.

'Coulomb' friction, i.e. friction whose magnitude is independent of velocity, has a well-deserved reputation for causing dynamic non-linearities in measuring instruments. Thus, an accelerometer in which Coulomb friction is present carries with it the limitations of resisting the motion until the acceleration force is sufficient to overcome the frictional force (dead spot) coupled with a dynamic response which cannot be described by a linear differential equation with constant coefficients.

On the other hand variable reluctance, linear-variable-differential transformer (LVDT) and variable capacitance pick-offs provide significant advantages in that sliding contact and friction problems are avoided and even small displacements are capable of being converted into suitable output signals if dimensional changes of the measuring instrument due to temperature are small.

6.4 Dampers

The use of dampers on accelerometers is necessary to provide means of controlling the response of the instrument to dynamic inputs. For high dynamic performance it is important to ensure that the damping is proportional to velocity and does not alter appreciably under environmental conditions experienced in flight. The characteristics of gas and oil damping systems, working in either the viscous shear or fluid displacement configuration, change appreciably with pressure and temperature respectively. Although the variation in viscosity of silicone oil with temperature is much less than for other oils it is still nevertheless large enough to be important in the accurate and reliable prediction of the dynamic performance of acceleration.

Other disadvantages, more obscure perhaps, but nevertheless important, associated with these types of damping systems are secondary mass effects, viscous effects superimposed on shear damping and compressibility effects.

Many present day accelerometers incorporate methods for control of the damping medium by locating diaphragms in the fluid. They allow for the expansion and contraction of the fluid with change of temperature and control the damping over a large range of temperature.

For high dynamic performance the device that satisfies most requirements for velocity damping under all conditions is the eddy current damper. Difficulty may be experienced in trying to incorporate this type of damping due to the size of magnets required but the rewards of achievement are worthwhile in that the damping force is:

(a) for all practical purposes, proportional to velocity, and

(b) changes only slightly with temperature and is independent of atmospheric pressure. Furthermore no links or lever arms, which can introduce friction or backlash into the system, are required.

6.5 Other types of accelerometer

The basic ingredients of mass-spring accelerometers have been described. However, many other groups, employing a wide range of basic detector-transducer principles, are available. Since a text-book rather than a chapter would be required to cover them all the discussion has been confined to those characteristics which make their behaviour different to that of the mass-spring type.

The most common of these types employ strain gauges, both unbonded and bonded foil types, piezo-electric and piezo-resistive and the so-called vibrating string device.

6.5.1 Strain gauge

Strain gauge accelerometers exist in many configurations and because of their small size and the fact that they have infinite resolution they are well suited for flight test work. They can be energised by ac or dc voltages and although their outputs are on the low side, integrated circuit components can be incorporated to provide a high output. Damping is usually by oil and response is available to high frequencies. They are available over a wide range of acceleration.

6.5.2 Piezo-electric

The basic element in a piezo-electric accelerometer is a block of crystalline material which, when subjected to mechanical strain in a preferred axis, is capable of generating an electrical potential. These transducers are generally limited to measuring dynamic inputs as the potential developed by an initial application of strain is only held until the charge finally leaks away. As it is difficult to incorporate damping mechanisms use is restricted to the flat part of the modulus over the frequency spectrum. In most cases this is no hardship as natural frequencies extend to many thousands of hertz and although the damping is low it is not zero. They have the advantage of being very robust and are obtainable in many ranges.

6.5.3 Piezo-resistive

Piezo-resistive accelerometers use semi-conductor strain gauges bonded to a flexural spring supporting a seismic mass. The piezo-resistive device consists of a single element cut from a crystal of silicon. The material possesses a very much higher piezo-resistance characteristic (change in specific resistance with strain) than do metal strain gauges which depend mainly on dimensional changes to obtain a change in resistance. Unlike piezo-electric devices they require electrical excitation and they respond to dc or zero frequencies. Another characteristic of piezo-resistive transducers is that they have low output impedances which avoid noise problems particularly where long cable lengths are required.

Piezo-resistive accelerometers are ideal tools for measuring vibration and transients. For these applications they provide significant advantages in high frequency response over their metal strain gauge counterparts and their outputs are significantly higher.

6.5.4 Vibrating reed

The vibrating string device is considered to be a very accurate device for operational use. Its design is based on a very clever concept. A mass is supported by two flexible tapes that are caused to vibrate with frequencies proportional to the tensions applied. Both tapes have a preset tension. Under acceleration, however, the mass causes an increase of tension in one tape and a decrease in the other. The frequency difference between the two tapes is proportional to the applied acceleration. As frequency, of all the physical parameters, can be measured most accurately this device offers high precision performance with great simplicity and reliability. Moreover, cross-effects can be reduced to a minimum because the tensions increase equally and hence the frequency difference is unaffected. However errors may be produced if the instrument is used in a severe vibrational environment.

7 CLOSED LOOP ACCELEROMETERS FOR FLIGHT TEST WORK

7.1 Operating principles of pivoted arm type

There are two main types of force feedback accelerometer, differing in the method of suspending the sensing element. They are the pendulous arm and the rectilinear types, schematic diagrams of which are shown in Fig.8. The former, which is by far the most common, may employ a pivot and bearing but for the highest precision a spring hinge is used as in this illustration. The mass is free to move along the sensitive axis and its displacement is measured by an inductive or capacity pick-off. The force coils which are attached to the mass move in gaps of the permanent magnets. The working of the force feedback system can be seen from Fig.9. If an acceleration is applied to the case along the sensitive axis the mass (pendulous arm) will move from the null position relative to the case. This produces an emf in the pick-off coil which is roughly proportional to displacement. After amplification and phase sensitive rectification this signal is applied to the restoring coil which reacts with the permanent magnetic field to return the mass towards the null position; the actual final displacement is inversely proportional to the amplifier gain. If the closed loop system has a high enough gain, the stiffness of the suspension is negligible and, under steady state conditions, the acceleration force is balanced entirely by the electro-magnetic restoring force. It has already been shown that the sensitivity is practically independent of changes in loop gain, and provided both the permanent magnet and the mass are stable the current in the restoring coil will be accurately proportional to acceleration. Usually, the current is converted to a proportional voltage by means of a precision readout resistor.

7.1.1 Servo loop characteristics

Many commercial instruments employ high loop gains to reduce the effects of residual forces, friction and cross accelerations and if little or no mechanical damping is present the modulus passes so close to the -1, J0 point on a Nyquist diagram that phase and gain margins are not sufficiently adequate and thus the system is unstable. In order to stabilise the system and at the same time obtain controlled or specially tailored dynamic response characteristics damping must be introduced into the system. There are two methods of introducing damping into force feedback accelerometers.

One is to mechanically damp the loop by filling the unit with a suitable oil or by employing eddy current damping if space permits. This method has the advantage that stabilising or active frequency - sensitive networks are either simplified or even avoided altogether.

The other method produces a velocity term by either introducing a velocity feedback coil into the closed loop circuit or by employing a phase-lead (differentiating) network. A phase-lead network has the property of reducing the overall phase shift (lag) of the system which prevents the modulus from coming too close to the critical point on a Nyquist diagram. Thus the gain is increased at high frequencies which results in a stiffer system in addition to improving the damping. In common with other types of servo systems for aircraft use the performance should be satisfactory if the phase margin is about 30° and the gain margin at least 6 db.

Although the pendulous arm types are easily the most common amongst high precision accelerometers, they are not necessarily the best suited for flight test applications. They are, in fact, susceptible to transverse accelerations impressed along the arm because as the mass rotates about the pivot or hinge the direction of the sensitive axis is changed slightly. To limit these effects systems are made very stiff so that the displacement per g is small. The fact that these instruments have now got high natural frequencies make them susceptible to high level vibrational environments which may drive the system into non-linear and unsymmetrical regions of the amplifier gain characteristics. Because of the importance of the effects of vibration on system performance, section 11 is devoted to this subject.

7.1.2 Sensitivity to angular accelerations

In general, a pivoted-arm linear accelerometer responds to torques impressed about and parallel to its axis of suspension as well as to linear accelerations acting on the mass. Consider an idealized pivoted-arm accelerometer of point mass $\underline{m}$ and length $\underline{\ell}$. Then the torques impressed on the device by angular acceleration $\underline{\ddot{\theta}}$ about the pendulum axis and by linear acceleration $\ddot{X}$ acting along the measuring axis are:

$$T_A = m\ell^2\ddot{\theta} \qquad\qquad T_L = m\ell\ddot{X}\cos\theta \ .$$

For many accelerometers $\theta \ll 1$, hence $\cos\theta = 1$, therefore $T_L = m\ddot{X}\ell$

Equating these torques gives:

$$\ddot{X} = \ell\ddot{\theta} \ . \tag{36}$$

In other words, the linear acceleration indicated by the accelerometer as a result of impressed angular accelerations about the pendulum axis is proportional to the latter and to the length of the pivoted arm. For example, our idealized accelerometer with the point mass located at 1 cm from its axis of suspension produces an output of approximately 1×10^{-3} g units when it is subjected to an angular acceleration of 1 rad/s^2 parallel to, and about its axis of suspension. This in itself is not serious as the error in the measurement of linear acceleration is small for such a large angular acceleration and it is not difficult to evaluate errors expected in these circumstances. Needless to say our idealized accelerometer would not respond at all to angular accelerations impressed about axes passing through the centre of the point mass.

However, it is not always convenient or possible to mount all the instruments close to the aircraft CG, or near to a specific point in the aircraft about which the particular measurements are desired. In this case, if the physical separation between the aircraft CG and the instrument is large then these types of accelerometer may show a high level of response to angular accelerations impressed about the CG.

Referring to Eq(36), it can be shown that the equivalent linear acceleration recorded by the accelerometer is proportional to the algebraic sum of the length of the pivoted-arm and the distance of the centre of the point mass length-wise along the pivoted-arm from the axis about which the angular acceleration is impressed. Denoting the distance from the pendulum axis to the axis of the impressed angular acceleration by ℓ_A Eq(36) can be restated in the form:

$$\ddot{X} = (\ell_A + \ell)\ddot{\theta} \ . \tag{37}$$

These are 'worst-case' or 'best-case', if $\ell_A = -\ell$, conditions, of course, in that the pivoted-arm lies length-wise on a radius from the axis of impressed angular acceleration and that the latter is parallel to the pendulum axis. Obviously, if the pivoted arm lies tangentially to a radius of this axis only centripetal accelerations associated with the rotational accelerations will be measured. For instance, our idealized accelerometer will produce an output equivalent to 3×10^{-2} g units if it is subjected to an angular acceleration of 1 rad/s^2 when it is located 1 ft (30 cm) from the CG under 'worst-case' conditions.

In addition to these facts, it is well known that pivoted-arm linear accelerometers respond to angular accelerations about the pendulum centre of mass. This is due to the fact that a practical accelerometer has distributed mass, and unlike the mechanical model of our idealized accelerometer, cannot be represented by a point mass without restriction when the device is subjected to accelerated angular motion.

To obviate difficulties arising from these facts Corey[19], presents general methods of designing and developing multi-axis assemblies of separate single-axis pendulous servoed accelerometers so that each of the separate accelerometers behaves like a simple point-mass accelerometer, and so that the locations of all the equivalent point-mass accelerometers are coincident.

Briefly, the technique is based on the fact that a pivoted body is insensitive to angular accelerations impressed about an axis passing through the centre of percussion of the pendulous mass. The centre of percussion of a compound pendulum is that point at a distance from the centre of suspension equal to the square of the radius of gyration of the pendulum with respect to the centre of suspension divided by the distance from the centre of suspension to the centre of gravity of the pendulum. In other words, the centre of percussion can be considered as the point where the mass is concentrated on a compound pendulum to make it behave as an equivalent simple pendulum whose length is equal to the distance of that point from the axis of suspension. The centre of percussion of a pendulous mass is located at infinity when the axis of the pendulum is at its centre of mass; as, for instance, in the case of an angular accelerometer. In the case of an idealized point-mass pendulum, it can be shown that the centre of percussion is located at the point mass. It is thus possible to design an instrument having any desired relationship between the position of the axis of the centre of percussion with that of the pendulous body.

Hence, if the axis of the centre of percussion is strategically located so that it coincides with the aircraft's CG position it is clear that our pivoted arm accelerometer is now insensitive to aircraft angular accelerations about the CG. Obviously a cluster of three strapped down accelerometers can have their equivalent point-masses coincident and located at a desired point. However, in view of the following considerations, it is questionable whether judicious placement of the cluster of transducers in this manner would entirely prevent corrections from having to be made for angular motion of an aircraft.

Prerequisites for the successful implementation of this technique demand that the CG of the aircraft must not only be known accurately in the take-off condition but that it must also be known during flight. Now, although the position of the CG is relatively easy to define fore and aft it is very difficult to define to a close tolerance vertically and, in addition, it moves forward or aft due to consumption of fuel, movements of crew or passengers, release of stores, and expenditure of ammunition. Furthermore, aircraft structures as a whole are elastic and hence the CG is never a physical point on the aircraft structure. Although neutral and manoeuvre points, which are related to specific CG positions, are used for datum purposes on certain flight tests, these relationships unfortunately still do not help to locate precisely the CG position at any given time.

Despite these sources of error, it is known that the CG movement in flight of some aircraft is of the order of 5 cm. Migration or uncertainty of the CG of this magnitude may therefore be small compared to the distance an accelerometer may have to be mounted from the CG. Where CG movements in flight of some aircraft, e.g. supersonic delta, are measured in tens of cm, implementation of these procedures will be restricted.

In short, realization of these procedures in practice is achievable in certain instances and the use of such a package of accelerometers with coincident effective centres of mass will minimize their response to angular accelerations. Although all types of strapped-down linear accelerometers require the effects of linear and angular accelerations to be separated in their output responses, pivoted-arm types with their inherent properties of physical pendulums are probably the only types capable of being designed so that they discriminate to a high degree against angular accelerations.

7.2 Rectilinear types

In the rectilinear type the mass is suspended by ligaments so that it is constrained to move only along the sensitive axis. This method of suspension makes the device practically immune to cross accelerations at low frequencies and hence these systems do not require to have high natural frequencies. Consequently, the effects of severe vibrational environments are considerably reduced. However, extremely high standards of engineering are required to produce this type and, as far as the author knows, none are commercially available.

7.3 Performance characteristics

Force feedback instruments of the pendulous arm type may be purchased commercially with guaranteed performance specifications. Their precision is far greater than that of the open loop type because they operate at almost null displacement and the force-balancing spring is of the electro-magnetic type depending only on the field of a permanent magnet which can be made very stable. Their accuracies are at least two orders better, the ultimate limitation being probably set by long term stability of the magnet.

Their outstanding features are high steady state accuracy and linearity over wide operating ranges, low sensitivity to temperature, long term stability and low cross-axis sensitivity. Moreover, by employing linear integrated circuits and hybrid micro-electronics, force feedback accelerometers can compete in size, weight and cost with open-loop types.

These performance characteristics minimize serious errors previously unresolved or just simply ignored when less accurate instruments were used. Although errors arising from temperature changes, bias discrepancy, drift of pick-off null point etc., may be significant in certain circumstances, they are generally not present to a first order in high precision units and furthermore standard tests can usually be used to assess and introduce, if necessary, any corrections to the measurements.

8 ANGULAR ACCELEROMETERS

8.1 The measurement of angular acceleration of aircraft

Angular acceleration is one of the most difficult parameters to measure in aircraft flight test work. The reason for this stems from the fact that the magnitudes of angular acceleration normally encountered on aircraft are exceedingly small. This is not surprising when it is considered that, in general, aircraft are not designed to execute high angular accelerations either from the point of view of available control power or from reasons of safety.

Measuring ranges of representative angular accelerations are within ±0.2 rad/s^2 for large aircraft and for fighters from ±0.5 rad/s^2 to ±5 rad/s^2. The latter range is usually required for aerobatics and spinning trials and if these two 'violent manoeuvre' cases are omitted the measurement of representative angular accelerations could be performed in many flight tests by instruments having a range in the region of ±1 rad/s^2.

8.2 Design characteristics

To illustrate the difficulties confronting the design engineer consider a pendulous accelerometer of point mass m and length ℓ. Angular acceleration around the pivot axis causes a torque due to the moment of inertia of the mass, while the restoring spring k permits a small motion which is measured for the instrument output. In other words, the instrument obeys Newton's first law for rotational motion, *viz.* $T = I\ddot{\theta}$, where T is the torque or moment, and $\ddot{\theta}$ is the angular acceleration.

Although this instrument is somewhat impractical because it also responds to linear accelerations, it is useful in exploring fundamental principles. Now the torque due to an impressed angular acceleration is balanced by the spring restraint and is defined thus:

$$k\theta = m\ell^2\ddot{\theta} \quad . \tag{38}$$

If θ_1 is the angular deflection of the pendulous mass system for unit angular acceleration (1 rad/s^2) then

$$k = \frac{m\ell^2}{\theta_1} \quad . \tag{39}$$

The undamped natural frequency of the system is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m\ell^2}} \tag{40}$$

and substituting the value of k into Eq(40) yields:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{1}{\theta_1}} \quad . \tag{41}$$

Now the undamped natural frequency f_n is important to the flight test engineer because it determines to a large extent the frequency range of interest the instrument can cover and it also determines the manner in which the instrument eliminates high frequency unwanted components in the input. For these reasons and to enable the instrument to cater for representative dynamic studies on aircraft a natural frequency in the region of 10 Hz is normally desired.

From Eq(41) it is seen that if $f_n = 10$ and with a full scale input of 1 rad/s^2 the angular deflection of the system is approximately 2.5×10^{-4} rad, i.e. less than 1' of arc. Even for a length ℓ = 3 inches, making the instrument at least 6 inches in length for a conventional accelerometer whose mass is a right circular cylinder, the linear movement for 1 rad/s^2 input is less than one thousandth of an inch. Clearly, the measurement of such small working displacements in the presence of uncertainties due to frictional torques and mass unbalance and dimensional changes due to temperature, precludes the possibility of designing an instrument to give accuracies in the region of 1%.

8.3 Proprietary instruments

To overcome some of these problems resort can be made to artifices such as:

(a) floated units employing paddles

(b) the measurement of the pressure generated within a fluid-filled column, and

(c) utilisation of the properties of a rate gyroscope.

One angular accelerometer which has been successfully used in flight test work for several years is constructed with a fluid in a circular channel serving as the accelerometer mass and a pivoted paddle immersed in the fluid. Inertial forces set up in the fluid by acceleration torques react on the paddle. The latter is attached to the housing by unbonded strain gauges which, in addition to measuring the magnitudes of applied torques, also provide the restraining torques on the inertial system. Because frictional torques and unbalance problems are virtually eliminated in this type of transducer, resolution is limited only by the noise level of the pick-off and associated circuitry. Several ranges are manufactured and the one of greatest interest to flight test engineers is the smallest, *viz.* ±1.5 rad/s^2. Despite the fact that its frequency is on the low side, approximately 4 Hz, with a damping factor of 0.7 this instrument is still capable of performing many useful measurements.

The use of fluids as inertial masses is also being currently exploited in transducers employing electrostatic feedback principles. The transducer consists of a circular column of fluid or a helical column of dense gas in which an electrostatic pressure motor is located. This force balance device measures the inertial torques of the fluid or gas produced by acceleration torques on the instrument by relating the output voltage to the applied torque or pressure. Ref.14 describes an instrument of this type for measuring physical parameters such as angular acceleration. An angular acceleration transducer employing this principle is potentially capable of providing a much higher degree of precision than other types.

8.4 The use of rate gyroscopes for angular acceleration measurements

One of the simplest methods for measuring angular acceleration consists of using a rate gyroscope and differentiating either by use of a coil attached to the instrument and moving in a magnetic field or by operating on the electrical output signal. Rate gyroscopes rely on angular momentum as opposed to inertia for measurement purposes and if the rotor has a reasonably large moment of inertia and a high spin rate angular velocities as low as 1' of arc s^{-1} can be detected.

The process of differentiation can be carried out either by inducing a voltage proportional to the first derivative of the angular velocity, as obtained from the induction coil, or by employing an operational amplifier with feedback to perform the mathematical operation on the electrical output. However, as will be shown, both these techniques have the same troublesome property.

For simplicity, consider a signal expressed in terms of sine waves of various frequencies. Any complex wave which represents the variation of a real physical quantity may be expressed as a constant term and the sum of a series of sine waves. Considering one component only of the sine waves this can be expressed as a voltage.

$$V_i = V \sin \omega t \quad .$$

Differentiation is achieved when the output is expressed as $\underline{V\omega \cos \omega t}$. This shows that the gain is proportional to frequency and results in the amplification of unwanted high frequency signals. Hence, considerable caution must be exercised before use is made of these techniques if the vibrational environment is severe. An advantage of these methods is that both angular acceleration and velocity can be obtained from one transducer.

Another method for measuring angular acceleration and utilising a rate gyroscope consists of supporting the rotor in two gimbals instead of one. Referring to Fig.10 it is seen that in addition to a single-axis rate gyroscope there is a second gimbal with elastic coupling about the input axis Y-Y. During constant angular velocity of rotation around axis Y-Y, the inner gimbal reacts in exactly the same way as a rate gyroscope, i.e. rotates around axis X-X. Under this condition the spring of the outer gimbal is not deformed, since the moment of the spring on the inner gimbal creates a rate of precession of the gyroscope equal to the impressed angular velocity, and hence moments acting radially throughout the inner gimbal bearing are zero. If the inner gimbal is now given an angular velocity as a result of an applied rate of change of angular velocity, i.e. angular acceleration, reactions on the inner gimbal bearings will cause moments about the input axis Y-Y, turning the outer gimbal with respect to the instrument case until the torque is balanced by spring $\underline{k_2}$. If a constant angular acceleration is applied to the instrument through the axis Y-Y the outer gimbal will lag behind until the spring k_2 is stretched to the point that it exerts on this gimbal the torque just required to give it the same angular acceleration as the case. Consequently, the outer gimbal will then be stationary with respect to the case and its displacement, in the opposite direction to that of the applied angular acceleration which produced it, can be read as an indication of the actual angular acceleration. Again, as in the former example, both angular velocity and angular acceleration are measured on one unit.

8.5 Force feedback angular accelerometers

Perhaps the closed loop type of angular accelerometer offers the best possibilities for determining angular accelerations. The use of these types of instruments means that problems of non-linear springs, mechanical hysteresis, temperature coefficient of modulus of elasticity, and spring deflection due to temperature are non-existent. Furthermore, they are well suited to the measurement of angular acceleration because they operate by direct measurement in contrast to open loop transducers which rely on indirect measurement in that the force or torque is deduced from the measurement of the elastic deformation of one of the force-bearing members. This direct measurement feature overcomes many of the aforementioned difficulties.

The operating principles of force-feedback angular accelerometers are similar to those of linear servo accelerometers. Under the action of angular acceleration, a torque is generated on the rotor system which tends to develop an angular displacement. As motion takes place the position error detector and servo amplifier generate a large feedback signal which is returned as current to an electrical torque generator. This continues until equilibrium is produced between the two torques. The final result is a current, or the voltage it develops across a resistor, proportional to the angular acceleration, accompanied by a minute displacement of the rotatable balanced mass.

The governing equations of typical force-feedback systems can be derived from Newton's law of angular motion which can be written

$$I \frac{d^2\theta}{dt^2} = -k\theta \qquad (42)$$

where I = moment of inertia
$d^2\theta/dt^2$ = angular acceleration
k = spring constant
θ = angular displacement.

Thus the equation of motion of an idealized system is:

$$I\ddot{\theta} + b\dot{\theta} + k\theta = f(t) \quad . \tag{43}$$

This equation of motion represents a conventional second order system whose steady-state and dynamic behaviour correspond to the solutions obtained for the simple mass-spring system.

Proprietary transducers are obtainable with ranges in the region of ± 1 rad/s^2 and accuracies better than 0.1% FSD. Although the natural frequencies associated with these particular ranges are somewhat high, e.g. 30 to 40 Hz, implying poor noise rejection, some manufacturers provide units with lower frequencies if measurements are to be made in severe vibrational environments. From past experience, and from a thorough investigation of previous work in this field, the seismic system should be a right circular cylinder with the input axis along the centre line of the cylinder. This method of symmetrical construction enables the instrument to reject unwanted interfering inputs such as angular velocities.

9 STEADY STATE CALIBRATION TESTS

9.1 Linear accelerometer tests

Calibrations of linear accelerometers are normally expressed in g units, 1 g being the acceleration in terms of the local terrestrial gravity. Three methods are commonly used for calibrating accelerometers. These are the tilting test, the added weight test, and the centrifuge test.

9.1.1 Tilting tests

One of the most convenient ways to calibrate an accelerometer is by the tilting test. The instrument is rotated about a horizontal axis at various angles relative to the local gravity vector. This method is suitable for low range instruments as they can be inclined through +1 g, zero and −1 g positions to obtain any fractional increment in this range.

However, there are two important sources of error. First, the sensitive axis may not be parallel or perpendicular to some reference face of the instrument because of manufacturing tolerances. In certain instances small misalignments of the order of 30' of arc may introduce errors of the order of 1×10^{-2} g units at a scale reading of zero g. By conducting a few preliminary tests it is possible to determine the total axis error. Suffice it to say that any misalignments between the measuring system and the appropriate mounting faces should be carefully noted to avoid faulty positioning of the installed instrument in the aircraft.

The other important general factor is the problem of ensuring that cross-coupling errors are correctly accounted for, particularly on an accelerometer which relies on the displacement of a mass. Cross-coupling is a change in output due to a component of acceleration perpendicular to the measuring axis. A fundamental feature of the tilting test is that for every position except alignment of the sensitive axis with the local gravity vector, i.e. ± 1 g, cross acceleration is present. The effect of this interfering variable is illustrated by a damped mass-spring system mounted on a tilting table as shown diagrammatically in Fig.11. It is clearly seen that the mass swings in a small arc when the system is subjected to acceleration forces. It can be shown that the output is of the form $g \sin(\theta \pm \alpha)$, to a high degree of approximation, where the sign of $\underline{\alpha}$ is dependent on the direction of the cross component along the arm, and its magnitude is equal to $\phi \sin\theta$, $\underline{\phi}$ being the full scale movement of the mass in degrees. The calibration curve resulting from tilting an accelerometer through 360° is shown in Fig.11.

As the design of many open loop instruments is a compromise between a system having large displacements per g to minimise dimensional and hysteresis effects for ease and stability of measurement, and one having small displacements per g to minimise cross-effects, it is inevitable that the mechanical deflection of the instrument's mass has well defined limits. In some practical arrangements this has resulted in the term $\underline{\alpha}$ being highly significant and, in one particular instance, produced an error of the order of 5×10^{-2} $\underline{g}$ at 0.5 g.

If the aforementioned test had been carried out with the instrument positioned so that the two non-measuring axes were interchanged, cross-effects would have been considerably reduced. However, even with this configuration, cross-axis sensitivity may be important if the accelerometer is operating in the presence of cross-accelerations of the order of 2 or 3 times the rated range. From these considerations it is seen that accurate calibration is required to ensure that test results are traceable to known standards of measurement.

High quality test equipment is needed as, in some instances, the measurement of angles to seconds of arc may be required for the tilting test. For these measurements methods employing goniometers or dividing heads and optical systems using digital readout systems have to be used to determine the angular position of the tilting table.

9.1.2 Added weight test

Another method of calibration, which can be used in a few cases only, consists of adding weights to the seismic mass to produce a deflection equivalent to that normally caused by acceleration forces. For this purpose, attachment points are located on the seismic element so that specially designed weights of non-magnetic material can be added to the mass to simulate precise values of g units. Practical accelerometers have distributed masses and the positioning of the attachment points must be correctly selected if improper loading of the springs or link assembly is to be avoided.

On the other hand, the sensing element must not be exposed to a concentrated rather than a distributed load. If these loading conditions have been assessed and taken care of by the instrument manufacturer it is considered that the added weight test provides acceptable accuracy for initial calibration purposes. Such tests are also of great value for calibration checks in the field.

9.1.3 Centrifuge test

Steady state accelerations higher than 1 g can be obtained by the use of a rotating arm or centrifuge. Predetermined increments of steady centrifugal forces can be imposed on the accelerometer by varying either the speed or the radius of rotation.

The indicated acceleration on the instrument may be checked by the following formula:

$$a = \frac{4\pi^2 n^2 r}{g} \tag{44}$$

where a = acceleration in g units
r = radius of the rotating arm in feet, corrected for mass position
n = revolutions per second of the rotating arm
g = local value of the earth's gravitational acceleration.

Since the test is performed in the earth's gravitational field, judicious placement of the instrument is necessary to reduce cross-coupling effects to the minimum. Comparison between the values obtained from tilting tests over ±1 g is a guide to the quality of tests done on the whirling arm.

Points worthy of special attention to achieve the necessary accuracy are as follows:

(a) since the acceleration is proportional to the square of the angular velocity, speed measurement and regulation must be adequate to provide the required accuracy,

(b) the radius of rotation should be large enough so that little or no correction is required for any changes that may occur in mass position due to the impressed acceleration,

(c) the axis of the rotating arm should be truly vertical so that a cyclic component of the earth's gravitational acceleration is not superimposed on the steady state acceleration generated by the rotating arm,

(d) vibrational noise must be within acceptable limits.

When it is desired to calibrate accelerometers up to 10 g with absolute accuracies better than 1 part in 10^{-3}, then a very precise and sophisticated whirling arm is required. These are usually so expensive to construct that they exist only in the principal centres for the evaluation of inertial guidance components. An example is the British Aircraft Corporation centrifuge at Stevenage, England, which is stated to calibrate accelerometers up to accelerations of 12 g with an absolute accuracy of acceleration measurements of 1 part in 10^5.

9.2 Angular accelerometer tests

Calibrations of angular accelerometers are normally expressed in rad/s^2, 1 rad/s^2 being normally taken as unit angular acceleration. It is difficult to devise experimental techniques for the production of precise values of steady state angular accelerations. Two methods can be used to calibrate an instrument - static and dynamic. The static method is accomplished by positioning weights at accurately known radii of gyration and observing the output when the weight exerts a purely vertical force on the measuring device. As, however, dynamic tests were probably necessary in the first instance to define the moment of inertia of the rotor it is debatable whether this type of test is capable of providing the desired accuracy. In dynamic testing the instrument is subjected to sinusoidal angular accelerations.

9.2.1 Added weight test

Obviously, the added weight test can only be accomplished when access can be gained to the rotor. For this type of test it is usually necessary at some stage to define the moment of inertia of the moving system. The moment of inertia depends on the distribution of mass elements and it is a difficult concept to define. If the rotor is a regularly shaped homogeneous specimen with accurately known dimensions use can be made of standard moment of inertia formulae. For rotors of irregular shape and non-uniform density this method would be time-consuming and not entirely adequate and hence resort would have to be made to dynamic methods. There are several types of dynamic methods for measuring moment of inertia; compound pendulum, bi-filar pendulum and torsional systems. The dynamic method consists of oscillating the undamped moving part of the instrument and measuring its period of oscillation. This, together with the spring force constant, determines the moment of inertia of the specimen.

Since it is difficult to eliminate damping or friction forces entirely and as the measurement accuracy depends on the accuracy of defining or measuring the deformation of an elastic force-bearing member it goes without saying that these measurements of the moment of inertia rarely approach the desired limits of accuracy. These considerations, coupled with the fact that the calibrating weights must be made to precise measurements and weights and, in addition, positioned on the rotor at an accurate radius of gyration, do not present the best possibilities for the basis of accurate calibrations of angular accelerometers.

9.2.2 Oscillatory tests

For this type of test electro-mechanical exciters or oscillatory tables, employing such methods as cross-heads or cams, fail to produce sinusoidal outputs at the large amplitude, in the region of ±30°, and low frequencies of the order of 0.5 Hz, needed to calibrate the low range accelerometers required for flight test work. One of the solutions to this problem is to employ a device similar in principle to an angular accelerometer but having little or no damping. The system is essentially a compound pendulum or a gravity controlled inertia device which, when released from an initial angular displacement, executes free oscillations.

Before mounting the instrument on the test rig platform it is imperative to evaluate the effects of unwanted inputs such as linear accelerations and angular velocities along and about all axes.

Angular position and angular velocity pick-offs are fitted to the test rig and their outputs together with that of the instrument are recorded over several cycles on a multi-channel oscillograph recorder. This procedure is repeated with suitable initial deflections of the pendulous device so that range, calibration factor, and linearity may be inspected. The peak applied angular acceleration is obtained from the product of the initial angular displacement and the square of the frequency. The required accurate knowledge of time or frequency is obtained by recording the output of a master-tuning fork or a crystal-controlled clock.

Two factors which can contribute errors to the test results are the transient input (complementary function) and the amount of damping present in the test rig.

Assuming that the instrument constitutes a second order system, as many of them are, and that it is being subjected to sinusoidal angular accelerations then the conditions existing at the moment of release of the deflected oscillating beam are shown in Fig.12, where it is seen that the applied angular acceleration is at a maximum. At this time, i.e. $t = 0$, the steady state output of the instrument should have a value corresponding to point $\underline{A}$, lagging the input by angle $\underline{\alpha}$, because of the damping force. This resisting force coupled with the inertia of the system results in the instrument needing time to catch up before it can swing into regular phase relationship with the applied angular acceleration. If the dynamics of the instrument and test rig are linear and the impressed frequency is much less than the instrument's natural frequency and $\zeta < 1$ it can be shown that the output is of the form:

$$\cos(\omega_i t - \alpha) - \frac{\cos\alpha}{\sin\phi} e^{-\omega_n \zeta t} \sin\left(\omega_n t\sqrt{1 - \zeta^2} + \phi\right) \tag{45}$$

where ζ = damping factor relative to critical

ω_i = impressed frequency (rad/s)

ω_n = undamped natural frequency (rad/s)

$\alpha = \tan^{-1} \dfrac{2\zeta N}{1 - N^2}$

$\phi = \tan^{-1}\sqrt{\dfrac{1}{\zeta^2} - 1}\ \left(\left[\omega_n^2 - \omega_i^2\right] / \left[\omega_n^2 + \omega_i^2\right]\right)$

and $N = \omega_i/\omega_n$.

The first term of the above expression represents the output of the instrument under steady conditions (i.e. some time after the release of the beam). The second term represents a transient which disappears comparatively rapidly, but which exists for a short time after the initial deflection. If it were possible to initiate deflection of the test rig so that the input had a value $\underline{\cos\alpha}$ at the instant the instrument was subjected to the impressed angular acceleration there would be little or no transient. The instrument would be in its correct (i.e. its final) phase relationship with the test rig. In all other cases, however, the transient term will not be zero.

From the above expression it was found that a linear second-order system, with $\zeta = 0.6$ achieves a correspondence to within a tolerance of 2% in modulus in approximately $2\pi/\omega_n$ seconds. In other words, with this damping the error becomes negligible at the beginning of the first recorded negative half-wave if the natural frequency of the instrument is ten times higher than the impressed frequency.

Fig.12 shows schematically the response of an angular accelerometer when the impressed frequency is half of the natural frequency of the accelerometer which has a damping factor $\zeta = 0.6$.

So far it has been assumed that the oscillating beam executes simple harmonic motion. Since frictional forces are always present the free vibrations decay and the mechanical motion takes the form of a damped sinusoid. It has been found from experience that if the bearings on the test rig are of first class quality and suitably lubricated and if the beam is of large inertia compared to that of the instrument the damping forces present are predominantly of the viscous rather than the coulomb friction type. Thus the damping force is approximately proportional to velocity and of the form $\underline{-b\dot{\theta}}$.

In many practical test set-ups the value of $\underline{\zeta}$ of the test rig is of the order of 0.01 or less. It can be shown that if ζ is $\ll 1$, a convenient rule of thumb method to correct the calculated angular acceleration input derived from the initial displacement and the recorded frequency consists of multiplying it by the factor $(1 - \zeta^2)e^{-2\zeta^2}$. With damping factors of the order quoted the errors are insignificant. For frequency response characteristics it should also be noted that the phase shift of the output should be retarded by $\underline{2\zeta}$ radians.

However, in circumstances where both the transient input and damping factor appreciably affect the results a practical and accurate method has been evolved to give final corrected results. The method is based on the fact that the wave-forms, after the transient has died out, are exponentially attenuated sine waves. A rough estimate is made of the time when the transient ceases to be of consequence and subsequent peak values are then plotted as ordinates and cycles as abscissae on log-normal graph paper. The resultant graph should be a straight line and from extrapolation of this line the true first peak value can be determined as if no transient existed. Alternatively, a plastic curve, having an exponentially shaped edge, can be superimposed on the recorded traces to achieve the same purpose.

10 DYNAMIC CALIBRATION

During flight testing it is frequently necessary to measure the oscillatory behaviour of an aircraft. The oscillations may be pilot induced, or may result from flying through turbulent air.

In order to measure these oscillations successfully and accurately, instrumentation systems based on sound dynamics in the characteristics and relations of the transducer, intermediate circuitry, and recording medium are required. Such systems must have predictable transfer functions under various input amplitudes and frequencies and environmental conditions of vibration, temperature and pressure.

It has been shown that linear second-order systems can be completely characterized by two fundamental parameters; the undamped natural frequency and the damping factor relative to critical.

From dynamic considerations alone, paying no attention to other requirements, two methods are favoured for selecting a transducer on the basis of its natural frequency and damping factor. The first requires a damping factor which endeavours to combine the widest bandwidth, consistent with an error not greater than say 1% in the modulus, with a phase shift increasing linearly with frequency so as to avoid destroying the time relationship existing between the fundamental and its harmonics. Thus, a complex waveform, consisting of the summation of many frequencies within the transmission range of the instrument, can be recorded without significant distortion. To meet the condition of the modulus requires a damping factor of 0.64 critical and the phase shift case requires 0.75 critical. However, as a damping factor of 0.7 introduces very little distortion of the harmonics in a complex waveform this value is usually chosen. The natural frequency is usually selected on the basis that it is in the region of ten or twenty times higher than the highest frequency signal of interest and if aliasing is known to be negligible and the vibration environment is not too severe.

The second specifies a natural frequency many times higher than any signal to be recorded and a damping factor of sufficient value to prevent mechanical damage and saturation being inflicted on the transducer due to resonance or shock. An electrical low pass filter can now be inserted between the instrument and the recording medium to dominate the system's dynamic performance. Tailoring the transfer functions in this manner so that the outputs of all the quantities being measured have practically the same dynamic characteristics is useful in flight tests, particularly where cross-spectra are to be computed. As will be shown later, this method can only be used when the vibrational noise is at a low level, or, the accelerometer has a much larger range than the quantities being investigated.

The single-degree-of-freedom mass-spring system discussed in section 2 is idealized by the mechanical model shown in Fig.1. For the ideal transducer the spring force and the viscous resistance are linear, i.e. the spring force is directly proportional to the change in length and the viscous resistance is directly proportional to the velocity. Even though it may be possible to describe the characteristics of a real transducer by a simple mechanical model and the associated differential equation, the spring constants and damping coefficients still must be determined experimentally. In other words, even if the performance characteristics are available in published form or set forth in a theoretical analysis it is imperative to ensure that the assumed conditions really apply. In this field of work there is no substitute for test data.

The actual characteristics of any system can only be determined in the laboratory by dynamic tests. The type of test commonly used for this purpose is the analysis of the response of the system to a sinusoidal input at differing frequencies.

10.1 Sinusoidal forcing functions

The main advantages of this method are:

(a) The sinusoid is the only periodic waveform in which the amplitudes of the components of all frequencies except the fundamental are zero. Naturally, a sine wave has only one bar in its spectrum. This feature eliminates the frequency distortion which could arise from an excitation consisting of a number of sinusoidal variations at different frequencies. This distortion of the composite output signal arises when the characteristics of the instrument are such that each component is displaced along the time axis by a different amount. Some flight testing techniques, of course, may require tests to obtain a knowledge of the dynamic response properties of the instrument when subjected to excitation with complex waveforms.

(b) The system's response to sinusoidal excitation yields information on the effects of coulomb friction, backlash, and damping that is not proportional to velocity if these features are present in the system.

(c) A basic property of the sinusoid is that any derivative with respect to time is the shape of a sine wave. Clearly, this is a distinct advantage as the test measurements include amplitudes and their time derivatives. If the parameters of the system are linear then sinusoidal inputs will always produce sine wave types of output.

(d) It allows a precise comparison between the response at very low frequencies and the steady state response, thus providing a criterion of the reliability of the dynamic test data.

(e) It can test the system for multiple resonances by covering a wide frequency bandwidth.

The ideal periodic function generator produces pure sine waves of controlled frequency and amplitude, and the frequency response curves are determined directly from pairs of associated input and output functions. Values of the latter are obtained throughout the frequency spectrum and they define the frequency response function, which is a complex valued function of frequency such that:

$$A_{(f)} = |A_{(f)}|e^{J\phi(f)} \tag{46}$$

where $|A(f)|$ is the absolute value of $A(f)$ and is called the gain factor, and $\phi_{(f)}$ is the associated phase angle which is called the phase factor.

The gain and phase factors may be interpreted as follows. If the forcing function is a sinusoid, the response of a linear transducer will also be a sinusoid. Hence the shape and frequency of the output are identical to those of the input. However, the ratio of the response amplitude to the excitation amplitude is equal to the gain factor (magnification) $|A_{(f)}|$, and the phase shift between the response and excitation is equal to the phase factor $\phi_{(f)}$.

This means that if a linear second order transducer is subjected to an input, which can be represented as the sum of a number of sinusoids, then the output will consist of these same sinusoids modified only in amplitude and phase. Typically, the transducer senses and displays each sinusoid as if it were the only signal present. The output will consist of the sum of the sinusoids modified by the response function. Further, if the input is restricted to the flat part of the response curve and the system has a damping factor of 0.7, i.e. the phase lag is proportional to frequency, then in addition to the above-mentioned features the input and output will also have the same shape. Once the characteristics of a linear system are defined by steady-state sinusoidal excitation then the response output for any specified excitation can be determined by analytical computation. These fundamental features place sinusoidal excitation in a unique position with regard to dynamic calibration of transducers.

Many devices in the form of electro-mechanical shaking tables, compound and torsional pendulums, and large cantilever springs can be constructed to produce mechanical displacements of sinusoidal form to the required degree of symmetry.

10.2 Aperiodic input functions

Having once established the frequency response function by sinusoidal forcing function, other idealized inputs can be used for experimental dynamic calibrations and field checks. One of the more popular of these techniques is the analysis of the response to a step function input. This method can be relatively simple and straightforward if access can be gained to the system. It consists of determining the natural frequency of the instrument with zero damping. When damping has been added the damping factor is readily interpreted by observing the overshoot after a step function has been applied to the system.

The method is applicable only to systems which are under-critically damped (oscillatory), as they usually are; the percentage overshoot is a function only of the damping factor and is given by $100\ e^{(-\zeta\pi/\sqrt{1-\zeta^2})}$, Fig.5. Strictly, this technique can only be used if it is known, *a priori*, that the system can be described by a linear second order differential equation with constant coefficients.

Another method occasionally used to specify the frequency response of a transducer is to examine its behaviour in response to an impulse function. The Dirac delta function is a single impulse or 'spike' at time zero having infinite amplitude, zero time duration and unit area. Such a pulse has a constant energy frequency spectrum ranging from zero to infinity. Although delta functions do not exist in real life, functions with properties approaching these mathematical concepts can sometimes be achieved in practice.

The practical difficulties of impulse response testing are sometimes very severe because the impulse function may be difficult to achieve in practice and some compromise has to be made. For instance, it is important that the power spectrum of the impulse, within the frequency bandwidth of the transducer, is constant and a practical impulse might not give this. Another difficulty with this approach is that an extremely large amplitude must be achieved to generate enough power to make spectral components measurable, and such an amplitude would probably saturate the transducer and parts of the system under test and so drive them into non-linear regions.

On the other hand a step input contains all frequencies from zero to infinity and furthermore it has a power spectrum the amplitude of which is proportional to $1/\omega^2$. It is evident that frequency response characteristics of transducers, which invariably have frequency components concentrated at the low end of the frequency spectrum, can best be studied by step inputs rather than by impulse functions because most of the energy is concentrated at the low end of the frequency spectrum.

10.3 Electrical excitation

Force-balance systems are sometimes dynamically calibrated by inserting into the system an electrical signal which simulates a mechanical excitation. Although these closed loop response tests are potentially very accurate, it is desirable to commence with mechanical inputs to establish the transfer function so as to relate the adequacy of the results so obtained to the values produced by the simulation.

11 COMPARISON BETWEEN OPEN AND CLOSED LOOP ACCELEROMETERS

11.1 Background

The preceding chapters have laid the groundwork for an understanding of the fundamentals and principal performance characteristics of open and closed loop accelerometers. It is now convenient to compare the relative merits of these two types.

Until recently, the only types of accelerometer available were those in which the force of a spring balances the acceleration force imposed on a mass. To obtain high enough outputs and to minimize errors due to dimensional changes resulting from temperature variations and hysteresis of the elastic suspension, large working displacements are required. Now, for many practical designs, the working displacement of the mass-spring assembly must be restricted to very small values to avoid excessive cross-coupling because accelerometers have to solve vector equations in which direction is equally important as magnitude. Furthermore, accuracies are seldom better than 1 or 2% even when large working displacements are employed. Consequently, flight test engineers are frequently faced with making concessions in their demands for high steady state accuracy and low cross-effects.

Despite these shortcomings, transducers with large working displacements have one redeeming feature, mainly low cut-off frequency. It follows that the transmission range is confined to the low end of the frequency spectrum and hence the instrument's transmissibility to high frequency unwanted inputs, e.g. vibrational noise, is conveniently restricted.

Now, it has been shown that the closed loop accelerometer derives its usefulness from its ability to make precise measurements of acceleration with a degree of precision probably greater than that of any other acceleration measuring device. It would appear, therefore, that force feedback sensors offer a panacea for all the ills associated with the conventional type of transducer. However, many commercial units incorporate tight or stiff loops to lower the cross-axis sensitivity. The system is now responsive to inputs of a very much higher frequency content than hitherto, and significant errors can arise which would not have been experienced with open loop types. For instance, when the signal to be measured is either a vibration or is a steady state or low frequency signal amidst unwanted noise then two characteristics *viz.*, dynamic response and even low cross-axis sensitivity, may give rise to errors.

11.2 Vibrational noise problems

On many flight tests, instruments are required to operate in a vibrational environment where it is impractical to satisfactorily isolate them from the whole or part of the vibrational envelope. It is assumed, of course, that remedial measures such as searching for nodes amidst the complicated vibrating structure to find less noisy mounting locations had been adopted when trouble first arose from high level vibration inputs. Furthermore, a factor which can exercise a controlling influence in this area is the stiffness of the instrument mounting. The resonant frequency of the mount plus sensor should be as far removed as possible from the transmission range of the instrument.

G-levels resulting from representative aircraft vibration input patterns tend to increase with increasing frequency, especially over that portion of the spectrum say from 20 Hz to 100 Hz. Examples of two vibration envelopes, one severe the other moderate, are shown in Fig.13. In general, aircraft vibrations will lie somewhere in between those illustrated. Vibration levels as high as ±10 g (i.e. approximately 0.02in double amplitude at 100 Hz) are frequently encountered and their effect on the accelerometer must be carefully considered.

Although instruments are designed to withstand such levels of vibration without damage, the fidelity of low frequency signals may not be preserved in their presence. The two main sources of inaccuracy due to vibration and which are associated with servo transducers are saturation and rectification errors.

11.3 Saturation effects

The rated range of many of these transducers is defined by the linear operational limits of the amplifier. If these are exceeded at any part of the instrument's frequency transmission range the amplifier may saturate.

Consider, by way of example, an accelerometer with a basic loop frequency of about 100 Hz and a damping factor 0.3. If the rated range is 20 g the presence of vibration at the resonant frequency of 90 Hz, Fig.2, may cause the amplifier to saturate if the vibration amplitude exceeds 12 g.

This cyclic saturation can cause large errors and distortion to the signal of interest, no matter what method of extraction is used or electrical filtering employed. Furthermore, the higher the level of saturation to which the instrument is subjected, the more unrealistic and irrelevant the output becomes, yet it is impossible to detect this error if a low pass electrical filter has been incorporated after the accelerometer.

There are a number of methods for preventing the maximum torquing capability of the servo system being exceeded anywhere within the transmission range of the instrument.

One is to select a transducer of high range, say ±20 g, and damping factor of $\zeta = 0.7$ on the basis that vibration levels seldom exceed 15 g and there is no peak in the instrument's frequency response. This is reasonable in view of the fact that even the sustained level of acceleration in one direction, which is required to be measured and rarely exceeds ±1 g on many flight tests, coupled with a ±15g vibrating input will not approach the instrument's rated range value. This high range transducer, while selected for a particular example, is broadly representative of available types and notwithstanding its high range it is still capable of providing accuracies in the region of 5×10^{-4} g.

Another method is to introduce an active frequency-sensitive network into the servo loop to make the best compromise as far as possible between the conflicting design features of low cross-axis sensitivity and high noise rejection (i.e. absence of noise saturation). For example, a phase-lag network would permit the use of high loop gains to retain low cross-axis sensitivity at low frequencies and, by virtue of its attenuation at high frequencies, would reduce the possibility of saturation at the high end of the frequency spectrum. Great care would have to be taken to ensure that sufficient gain and phase margins were maintained to allow for unavoidable gain changes so that stability criteria were satisfied.

A method employed at the National Aerospace Laboratory, NLR, Amsterdam consists of mounting the accelerometer on a wooden board or anti-vibration mounts to isolate it from high frequencies.

Manufacturers claim that Neoprene bonded cork anti-vibration material can have a transfer function approximating to a linear low pass (second order) filter with natural frequencies and damping factors tailored to suit particular requirements. Although not enough is known about the behaviour of this material to recommend it as a method for inserting a low pass mechanical filter, it could make an excellent filter to attenuate high frequency inputs.

Probably the safest method to employ in most circumstances is to measure the vibration with a piezo-electric accelerometer and record its output simultaneously with that of the main accelerometers. The piezo-electric accelerometer measures vibration over a large part of the frequency spectrum and also short duration shock and power or Fourier spectrum analysis of the recorded information will reveal critical levels or discrete amplitudes.

11.4 Rectification effects

Rectification errors arise when vibrations are impressed on an accelerometer which is susceptible to cross-effects.

Referring to Fig.14, consider a pivoted arm type of instrument under the influence of an acceleration A_N along the sensitive axis; the arm is displaced by a small angle from null and the accelerometer will detect a fraction $\delta_C A_C$ of any acceleration A_C along the pivoted arm (i.e. perpendicular to the sensitive axis). The measured acceleration is given by the following expression, in which δ_C is the cross-coupling coefficient:

$$A_{meas} = A_N(1 + \delta_C A_C) \quad . \tag{47}$$

It follows that if A_N and A_C are components of a vibrational acceleration $A \sin \omega t$ acting at angle θ to the sensitive axis we have:

$$A_N = A \sin \omega t \cos \theta \quad \text{and} \quad A_C = A \sin \omega t \sin \theta \quad . \tag{48}$$

Therefore,

$$A_{meas} = A \sin \omega t \cos \theta(1 + \delta_C A \sin \omega t \sin \theta) \tag{49}$$

$$= A \sin \omega t \cos \theta + \delta_C A^2 \sin^2 \omega t \cos \theta \sin \theta \tag{50}$$

$$= A \sin \omega t \cos \theta + \tfrac{1}{2}\delta_C A^2 \sin^2 \omega t \sin 2\theta \quad . \tag{51}$$

The first term in the above expression represents the component of the vibration which the ideal instrument would measure and the second term, which is at a maximum when $\theta = 45^o$, represents harmonic distortion which alters the magnitude and shape of the output waveform thus increasing the probable value of error in modulus and phase measurement. For combined inputs, where the acceleration to be measured is of low frequency, and the vibration A is unwanted noise, a filter would normally be inserted. This would effectively remove the first term whose average value is zero but the second term whose average value is $\tfrac{1}{2}\delta_C A^2 \sin 2\theta$ represents a steady state error.

For a given arm radius r the closed loop natural frequency f_n is related to the static angular deflection per g of acceleration (δ_C rad/g), i.e. the cross-coupling coefficient, thus:

$$f_n = \frac{1}{2\pi}\left(\frac{g}{\delta_C r}\right)^{\frac{1}{2}} \quad . \tag{52}$$

If our 100 Hz and our 0.3 damped instrument has a length of pendulous arm $r = 3$ cm then $\delta_C = 2.8$ arc min/g $= 8.5 \times 10^{-4}$ rad/g and its steady-state cross-axis sensitivity is 8.5×10^{-4} g/g which is a very good figure indeed. However, the value of δ_C is related to the transfer function of the instrument and if the modulus is not flat but tends to rise, δ_C will have a corresponding increase and hence accuracy will be reduced at frequencies near the instrument's resonant frequency. For example, referring to Fig.2, the modulus for a damping factor 0.3 rises to a peak at the instrument's damped natural frequency (90 Hz) thereby increasing δ_C (cross-axis coefficient) by a factor of 1.8.

If our instrument is subjected to an oscillatory input in the region of 10 g amplitude, i.e. ½ peak-to-peak, at a frequency of 90 Hz and acting at an angle of 45° to the sensing axis, the steady component of error due to the rectification effect is approximately (2.1×10^{-2})g $\times 1.8$ or 4×10^{-2} g.

However, our lightly damped instrument is not representative of many existing types and is only used to illustrate 'worst case' conditions. If the damping, for example, were 0.7 the error would not exceed 2×10^{-2} g.

From these results it is seen that errors arising from rectification effects on high frequency pivoted arm types of accelerometers can be considerable and care must be exercised when these devices are used in the presence of severe vibrational environments.

The picture is considerably improved, however, when the case of a low frequency open loop type of instrument being subjected to a similar oscillatory input is considered. Choosing an instrument with typical values, e.g. $f_n = 20$ Hz, $\zeta = 0.7$ and $r = 3$ cm, and noting from Fig.2 that our 10 g amplitude, 90 Hz input, has now been attenuated by the transducer to the region of 1 g, the steady component of error is approximately 5×10^{-3} g. On the other hand, its susceptibility to steady-state cross-axis accelerations is too much. A value of 2×10^{-2} g/g is just not good enough for many flight test applications.

12 CONCLUDING REMARKS

Force feedback accelerometers are potentially capable of realising accuracies which are one or two orders higher than those of conventional mass-spring types. However, these high accuracies may not always be physically realisable in practice if high frequency systems are subjected to high levels of unwanted noise.

It has been shown that many limitations associated with conventional transducers have been minimised or eliminated by the use of servo techniques.

Some of the design considerations affecting steady state accuracy and system dynamic performance have been presented with the purpose of indicating practical solutions of measuring problems to the practising flight test engineer.

The performance of these instruments and in particular the way in which significant errors can arise have been discussed to aid the selection of the correct instrument for a particular task.

It appears that vibration still represents a significant factor when steady state or low frequency accelerations are measured in its presence, and improved means of isolating the instrument from aircraft vibration without degrading signal accuracy are urgently required.

If really high-grade rectilinear closed loop accelerometers become available at a not too exorbitant cost, then it is apparent that a wide scope exists in the aircraft flight test field for instruments of this type.

Until these appear, open loop types of accelerometers may provide more reliable results when measurements are performed in the presence of severe vibrational inputs.

A fall-out from space research is the three-axis accelerometer in which the mass is suspended in electromagnetic or electrostatic fields and which measures accelerations in three orthogonal directions. These non-mechanical springs permit acceleration measurements to be made in the region of 10^{-5} g and, in addition, in the case of electrostatic forces they are relatively insensitive to saturation.

Extensive programmes are currently being pursued to develop entirely new techniques for the measurement of acceleration. Many of these are just around the proverbial operational corner and if the reader wishes to make a study of the various types, Refs.14 to 18 make good reading.

Appendix 1

DERIVATION OF f_n, THE UNDAMPED NATURAL FREQUENCY

To derive the undamped natural frequency f_n of a mass-spring system consider a load of weight W suspended on a spring, Fig.15. AB is the length of the unstretched spring and AO is the length of the spring when stretched by the weight W. The spring constant k is the number of units of force required to stretch the spring a unit length. It follows that if $BO = \delta_{st}$ is the static deflection of the spring under load W when δ_{st} is equal to:

$$\delta_{st} = \frac{W}{k} \quad . \tag{53}$$

If the weight W is deflected from its equilibrium position at 0 and released it will oscillate about the position of equilibrium. In this simple study, as no forces other than the spring force, the earth's gravitational force, and the inertia reactance of the load are assumed to act on the system, the amplitude of oscillation remains constant. Such oscillations of the mass-spring system are called free or natural oscillations and they will be sinusoidal in nature.

Let Y be the peak amplitude and f_n the frequency of the oscillation. Thus the instantaneous value of the co-ordinate y of harmonic oscillation can be considered as a projection on axis oy of radius Y, rotating around the position of equilibrium of the mass at point O with constant angular velocity ω.

During one period P of the oscillation the vector rotates through 2π radians, therefore:

$$\omega_n = \frac{2\pi}{P} = 2\pi f_n \quad \text{rad/s} \quad . \tag{54}$$

The co-ordinate y will be equal to:

$$y = Y \cos \omega t \quad . \tag{55}$$

Successive differentiation of this equation yields:

$$\text{velocity} \quad \dot{y} = -Y\omega \sin \omega t \tag{56}$$

$$\text{acceleration} \quad \ddot{y} = -Y\omega^2 \cos \omega t \quad . \tag{57}$$

In Fig.15, assuming displacements below the equilibrium position are positive, and applying Newton's laws to the free body diagram as illustrated, we obtain

$$\sum F = m\ddot{y} \quad \text{or} \quad W - k(\delta_{st} + y) = \frac{W}{g}\ddot{y} \quad . \tag{58}$$

As pointed out previously

$$\delta_{st} = \frac{W}{k} \quad . \tag{59}$$

Therefore substituting this value of δ_{st} in Eq(58), we obtain

$$W - W - ky = \frac{W}{g}\ddot{y} \tag{60}$$

or

$$m\ddot{y} + ky = 0 \quad . \tag{61}$$

This is the characteristic differential equation which describes the motion of the system. Substituting the value of $\ddot{y}$ from Eq(57) and y from Eq(55), we obtain:

$$-mY\omega_n^2 \cos \omega t + kY \cos \omega t = 0 \quad . \tag{62}$$

Since $Y \neq 0$ by definition, hence

$$\omega_n^2 = \frac{k}{m} \quad . \tag{63}$$

Therefore

$$\omega_n = \left(\frac{k}{m}\right)^{\frac{1}{2}} \tag{64}$$

and

$$f_n = \frac{1}{2\pi}\left(\frac{k}{m}\right)^{\frac{1}{2}} \quad . \tag{65}$$

Appendix 2

DERIVATION OF ζ, THE DAMPING FACTOR RELATIVE TO CRITICAL

Of the two main parameters that completely define a linear second order system, e.g. ω_n and ζ, only ω_n has been defined. It is now appropriate to examine a system with a damping mechanism so that ζ, the damping factor relative to critical damping, can be derived.

Although damping forces can take many forms the most acceptable one, and indeed one of the few that can be analytically expressed, is that proportional to the first power of velocity. For this reason, only this form of damping is considered in this study.

In Fig.16, a weight W is suspended from a spring which is connected to a dashpot to provide viscous damping. The direction of the damping force is opposite to that of velocity and is written

$$F = -b\dot{y} \tag{66}$$

where the damping constant b is the number of units of resistive force per unit velocity of motion.

The free body diagram illustrates all the forces acting on the load when displaced a distance y below equilibrium and travelling downwards after the weight has been released from a deflected position. Note as before that k is the spring constant and that in the equilibrium position $k\delta_{st}$ is equal to W. The equation of motion under the above conditions becomes:

$$W - k(\delta_{st} + y) - b\dot{y} = \frac{W}{g}\ddot{y} \tag{67}$$

or

$$\ddot{y} + \frac{bg}{W}\dot{y} + \frac{kg}{W}y = 0 \quad . \tag{68}$$

Unlike the oscillations of an undamped mass-spring system, those of a damped system decay exponentially because of the energy dissipated in the damper. Therefore a solution of this differential equation is assumed to take the form:

$$y = Ae^{pt} \tag{69}$$

where A and p are constants.

Noting that:

$$\dot{y} = Ape^{pt} \tag{70}$$

and

$$\ddot{y} = Ap^2e^{pt} \tag{71}$$

substitute these values into Eq(68) to obtain:

$$Ap^2e^{pt} + A\frac{bg}{W}pe^{pt} + A\frac{kg}{W}e^{pt} = 0 \tag{72}$$

or

$$\left(p^2 + \frac{b}{m}p + \frac{k}{m}\right)e^{pt} = 0 \quad . \tag{73}$$

The desired solution must be such that the above equation be zero. Since e^{pt} cannot be zero then the term in brackets must be zero. Consequently:

$$p^2 + \frac{b}{m}p + \frac{k}{m} = 0 \quad . \tag{74}$$

The solution of this equation using the quadratic formula gives two roots:

$$p = -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} \quad . \tag{75}$$

Since m, b and k are always positive the type of root obtained is dependent upon the evaluation of the radicand. When the radicand is equal to zero, this value of b is called the critical damping factor b_c and is given by:

$$b_c = 2\sqrt{mk} = 2m\omega_n \tag{76}$$

where ω_n is the undamped natural frequency. A system with critical damping has the smallest damping possible for aperiodic motion and where the weight, after its release from an initial displacement, creeps back to equilibrium without oscillating.

The dimensionless ratio b/b_c is called the relative damping ratio ζ and is given by:

$$\zeta = \frac{b}{b_c} = \frac{b}{2\sqrt{km}} = \frac{b}{2m\omega_n} \quad . \tag{77}$$

When the radicand in Eq(75) is positive the motion is oscillatory and of a gradually subsiding nature and the damping is subcritical.

Since for practical transducers $\zeta < 1$, this case is the one of most interest to the instrument user and is also the one used for illustrative purposes.

REFERENCES

1 Draper, Bentley. Design factors controlling the dynamic performance of instruments. American Society of Mechanical Engineers Transactions, July 1940

2 Wilcock. Response of a simple accelerometer to a haversine change to constant acceleration. RAE Report Inst. 1449 (1945)

3 Bennett, Richards, Voss. Electronics applied to the measurement of physical qualities. R&M 2627 (1947)

4 - Statham Laboratories Inst. Notes, Statham Instruments, Inc., 12401 West Olympic Bldg., Los Angeles, California, 90064

5 R.W. Plumbly, R.J. Pitt, R.H. Evans. An assessment of a precision, viscous-damped force-feedback accelerometer. RAE Technical Note IAP 1135 (1962)

6 I. McLaren. Calibration methods for the accurate assessment of the static and dynamic performance of some flight test instruments. RAE Technical Note Aero 2914 (1963)

7 Michel Delattre. The OFERA low-g accelerometer. ONERA, 92 Chatillon, France

8 M, Gay. Accelerometre a grande sensibilitie. ONERA TP No.393 (1966), 92 Chatillon, France

9 R. Rose. Stability and control flight testing - some of the test instrumentation requirements. RAE Technical Note Aero (1967)

10 I.L. Thomas, R.H. Evans. Performance characteristics and methods of testing force-feedback accelerometers. RAE Technical Report 67183 (1967)

11 Dr. V.B. Corey. Electrically servoed transducers for flight control. Paper given at the International Aerospace Instrumentation Symposium, March 1968, Cranfield, England

12 K.H. Deotsch. The time vector method for lateral stability investigations. RAE Report Aero 67200 (1967)

13 - AGARD Flight Test Manual, Vol.II Stability and control, Vol.IV Instrumentation systems

14 W.R. Macdonald, C. King. An electrostatic feedback transducer for measuring very low differential pressures. RAE Technical Report 71022 (1971)

15 J.E. Miller. The PIPA (Pulsed Integrating Pendulum Accelerometer) E 934. Instrumentation Laboratory, MIT, Cambridge, Mass, USA (1960)

16 R.O. Bock. The vibrating string accelerometer. AGARD Conference Proceedings. Inertial Navigation - Systems and Components, May 1968

17 N.R. Serra. Technical Report on the quartz resonator digital accelerometer. Litton Systems, Inc., Woodland Hills, California, USA

18 E.J. Frey, R.B. Harlan. Application of inertial technology to airborne gravimetry. MIT, Laboratory Report, Cambridge, Mass, USA

19 V.B. Corey. Multi-axis clusters of single-axis accelerometers with coincident centres of angular motion insensitivity. Paper given at the 6th International Aerospace Instrumentation Symposium, March 1970, Cranfield, England

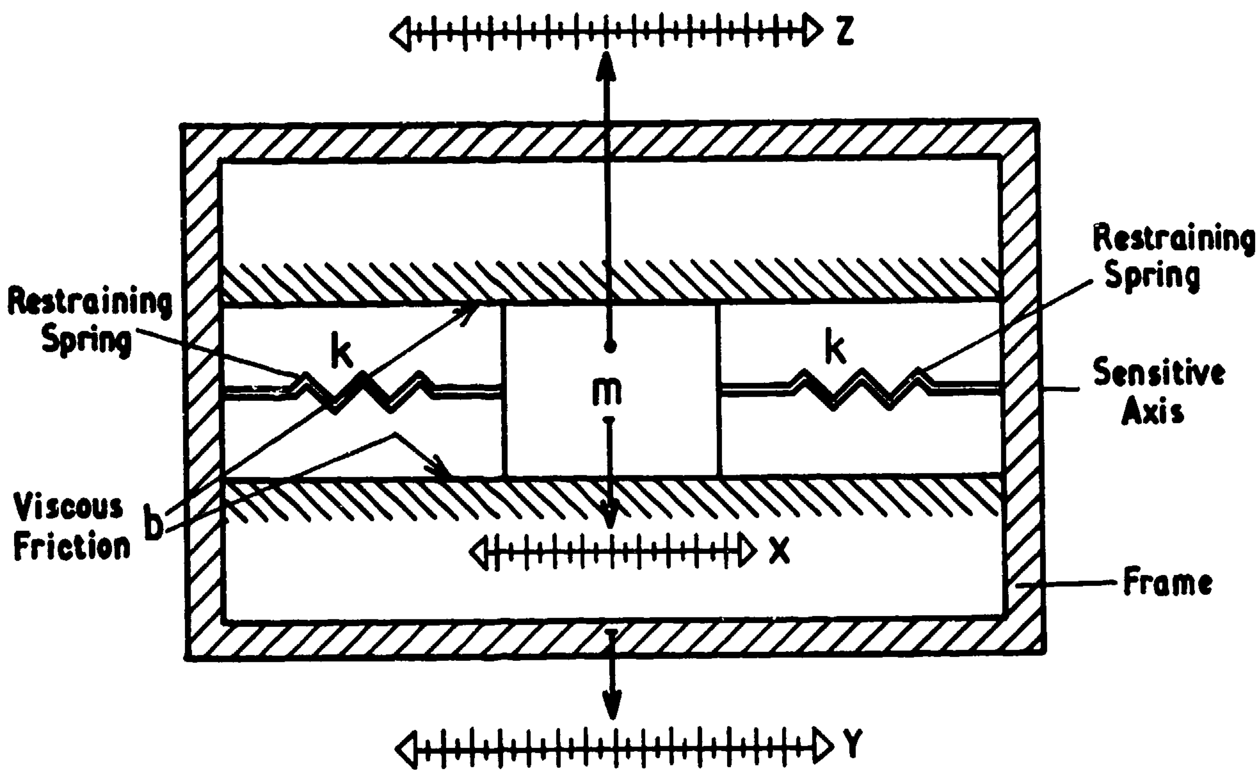


FIG. 1 SINGLE-AXIS ACCELEROMETER

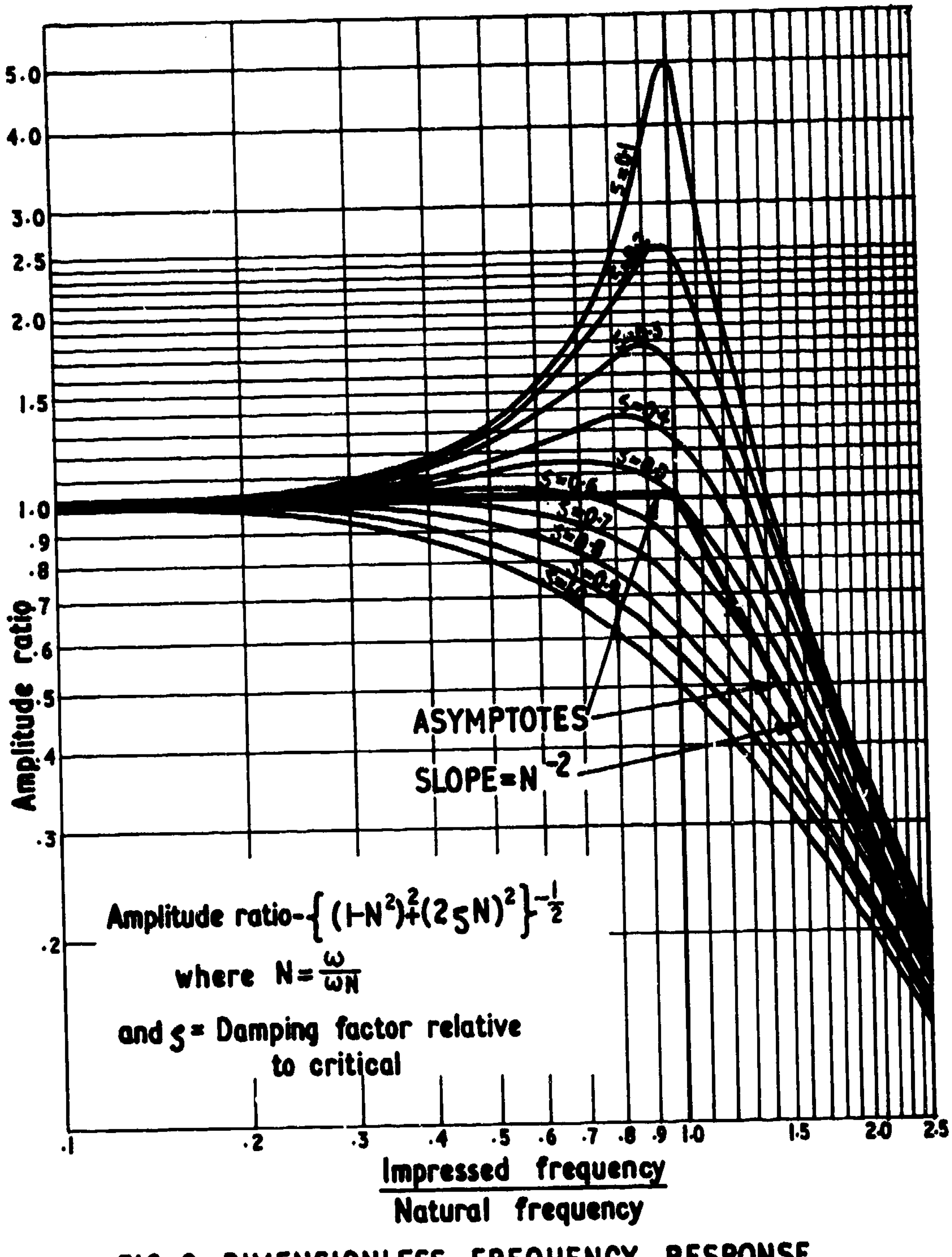


FIG. 2 DIMENSIONLESS FREQUENCY RESPONSE CURVES OF CONVENTIONAL SECOND ORDER SYSTEM.

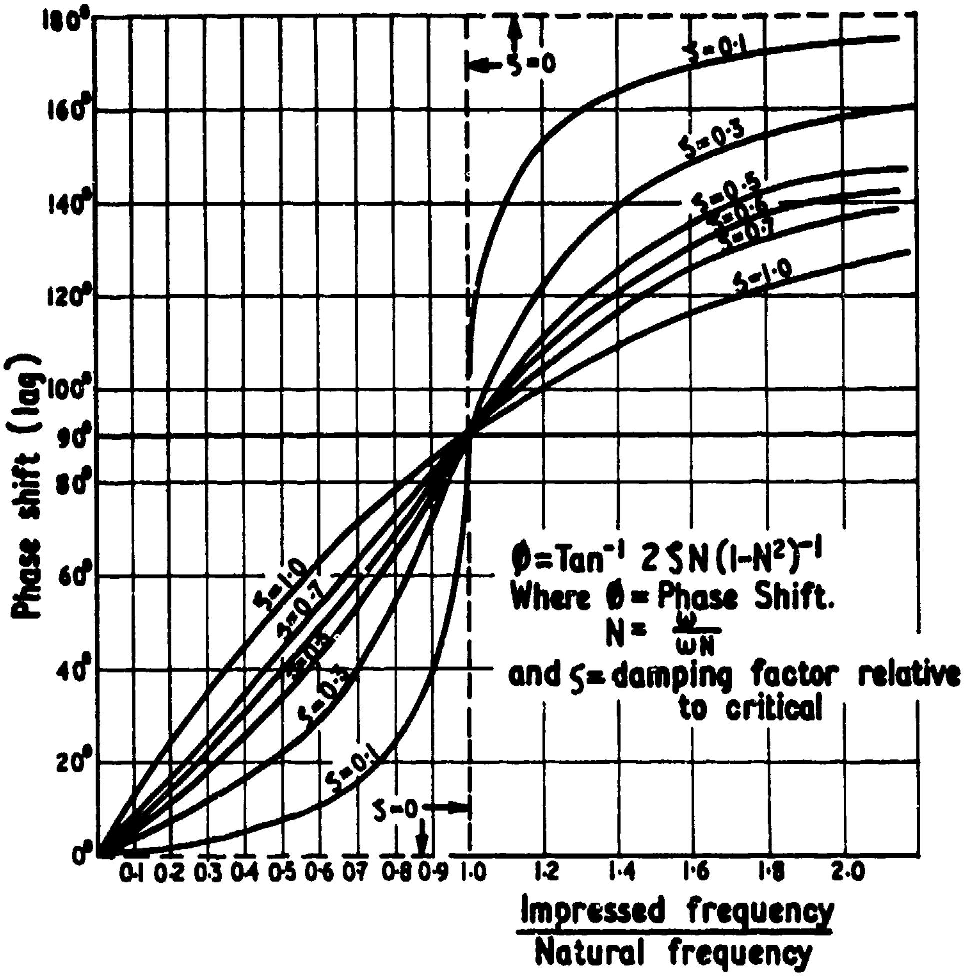


FIG. 3 VARIATION OF PHASE ANGLE WITH DAMPING OF CONVENTIONAL SECOND ORDER SYSTEM.

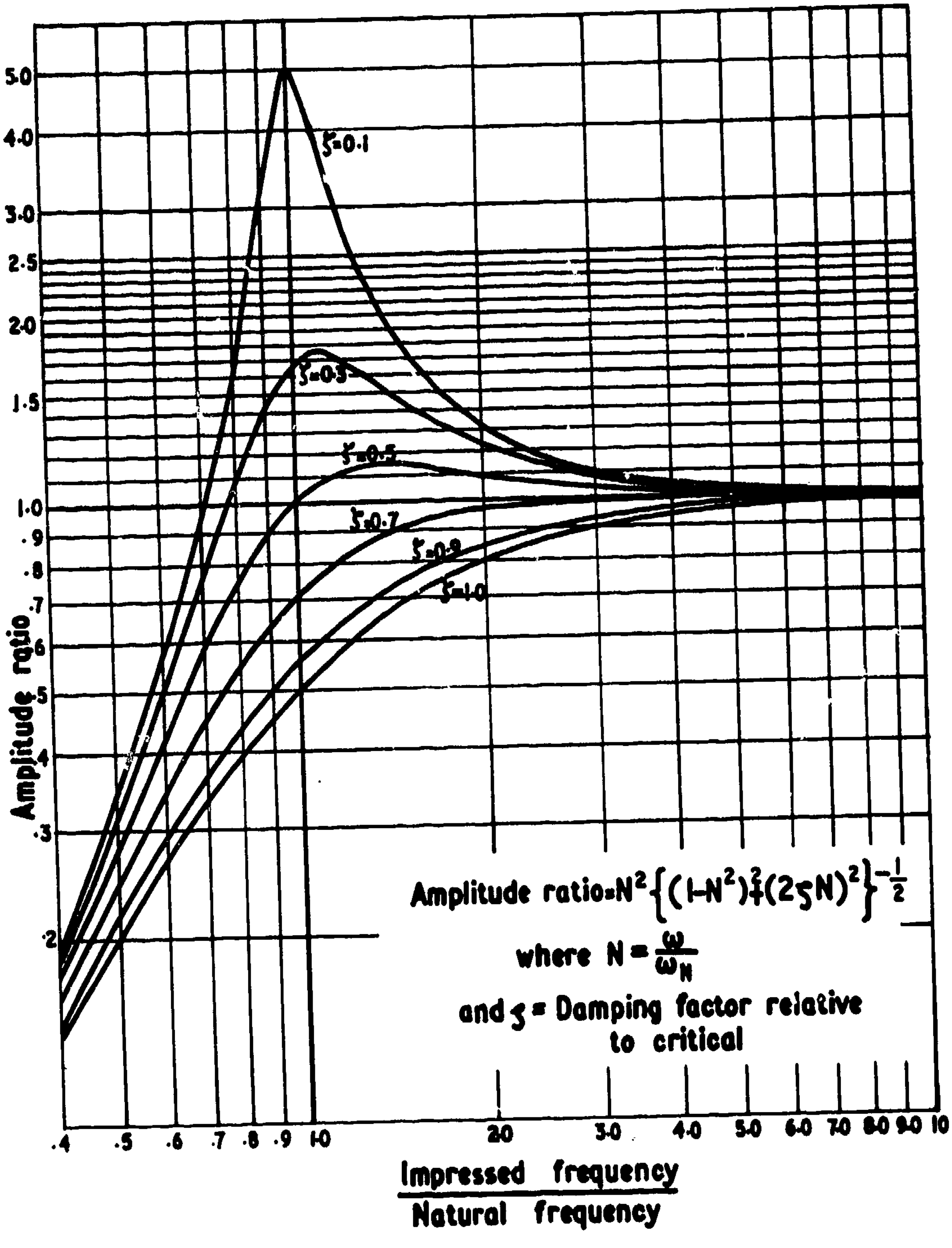
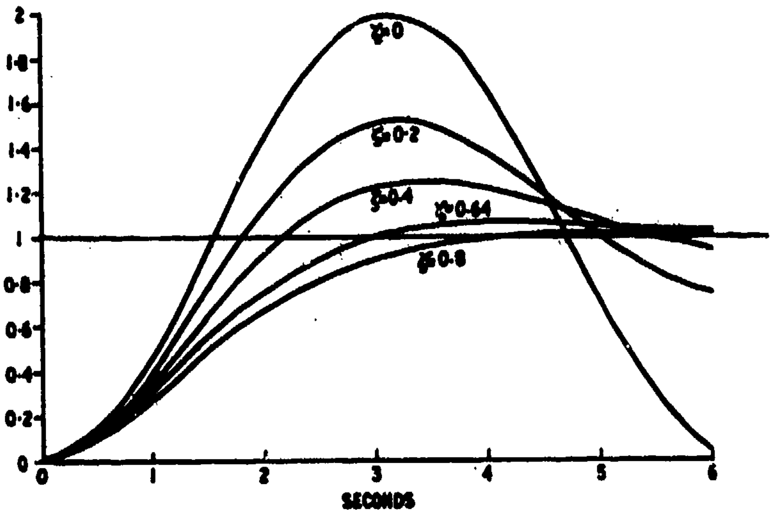


FIG. 4 DIMENSIONLESS FREQUENCY RESPONSE CURVES OF A SEISMIC PICK-UP



DAMPING FACTOR	PERCENT OVERSHOOT
0·00	100·00
0·05	85·4
0·10	72·9
0·15	62·1
0·20	52·7
0·25	44·4
0·30	37·2
0·35	30·9
0·40	25·4
0·45	20·5
0·50	16·3
0·55	12·6
0·60	9·48
0·65	6·81
0·70	4·60
0·75	2·84
0·80	1·52
0·85	0·629
0·90	0·152
1·00	0·0

FIG. 5 RESPONSES AND PERCENT OVERSHOOTS OF A SECOND-ORDER SYSTEM WITH VARIOUS DAMPING FACTORS TO STEP INPUTS.

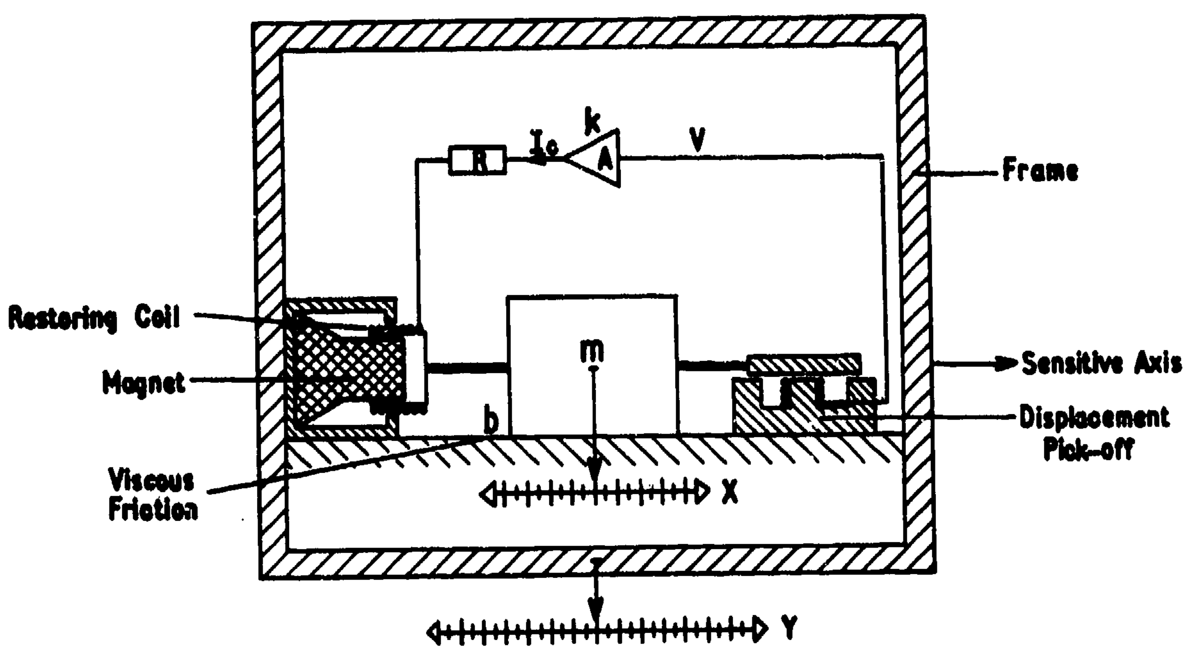


FIG. 6 SIMPLIFIED SERVO ACCELEROMETER

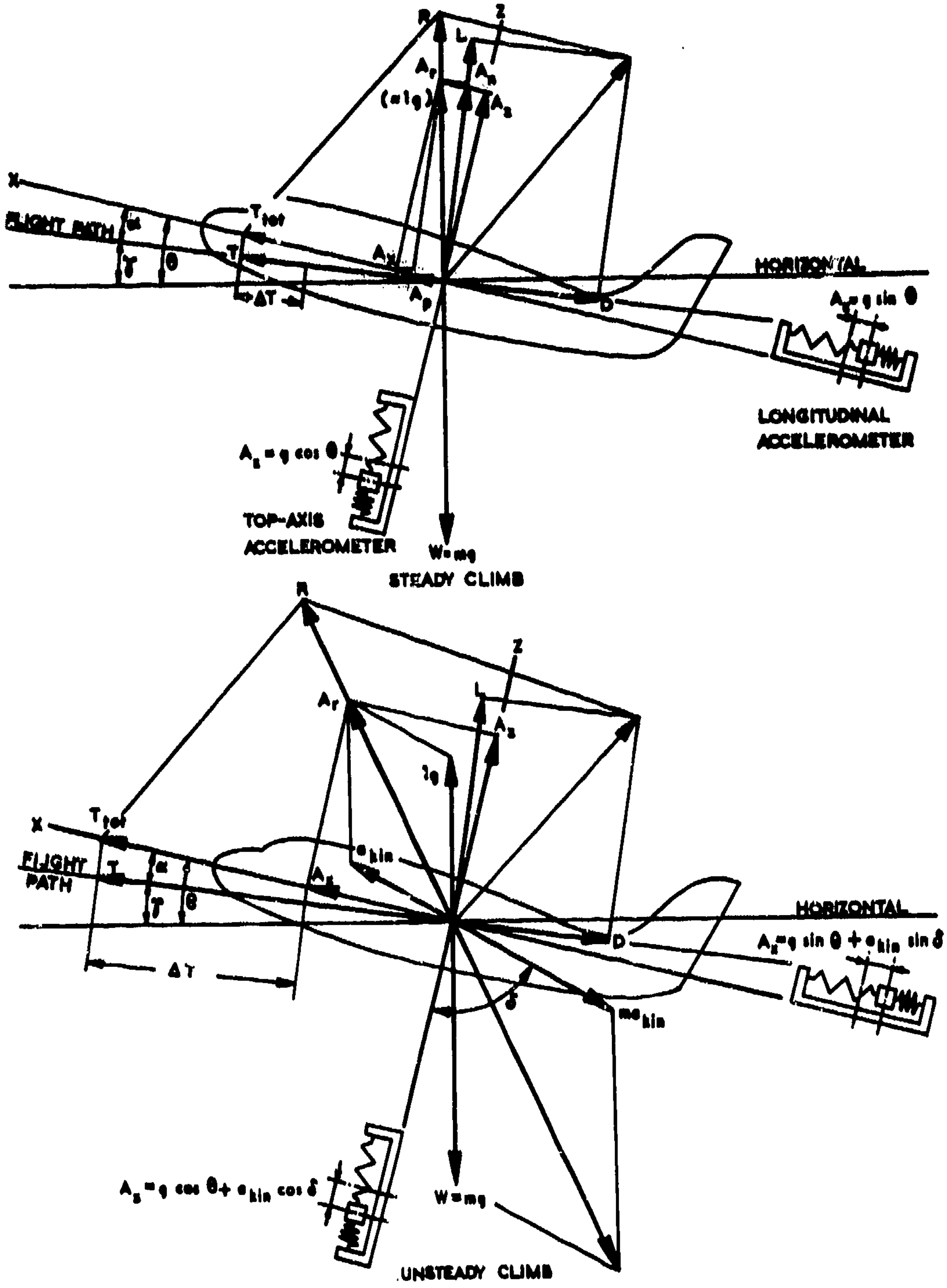
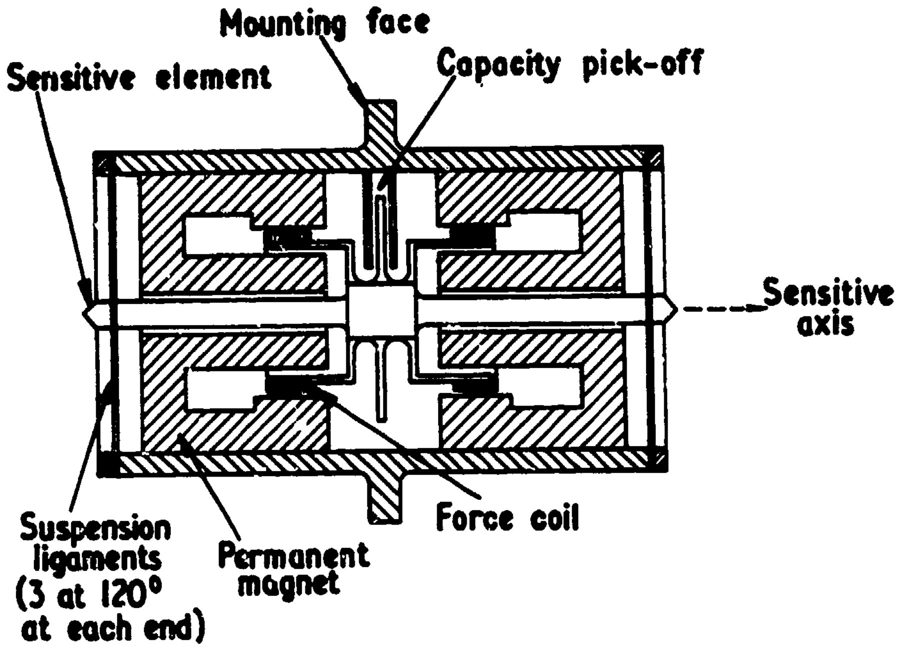
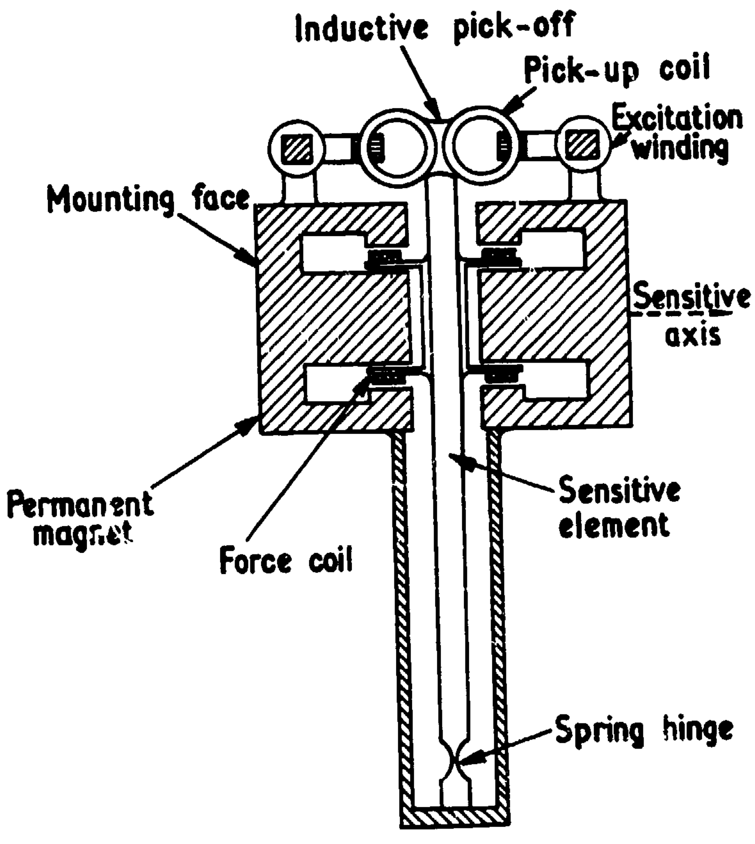


FIG.7 DIAGRAM OF FORCES AND ACCELERATIONS IN STEADY AND UNSTEADY CLIMB.



a. Rectilinear type accelerometer



b. Pivoted arm type accelerometer

FIG. 8 ACCELEROMETER SCHEMATIC DIAGRAMS

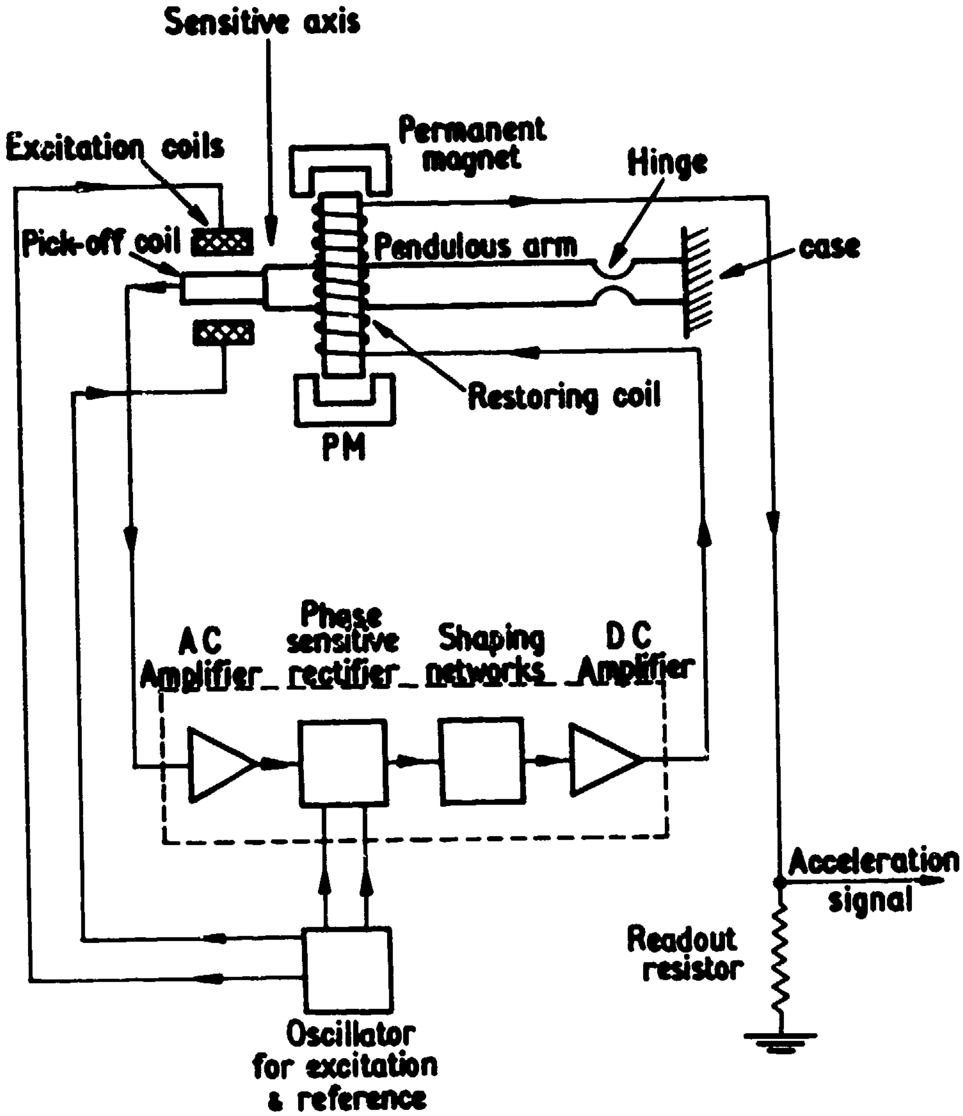


FIG. 9 CIRCUIT OF PENDULOUS ARM TYPE ACCELEROMETER

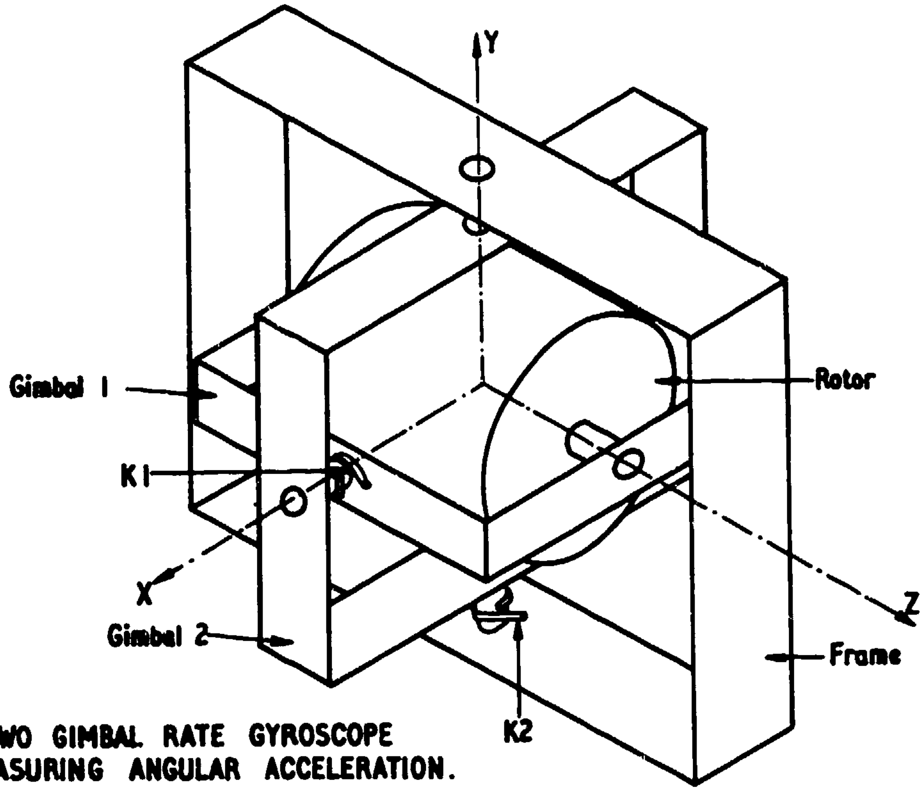


FIG. 10 TWO GIMBAL RATE GYROSCOPE FOR MEASURING ANGULAR ACCELERATION.

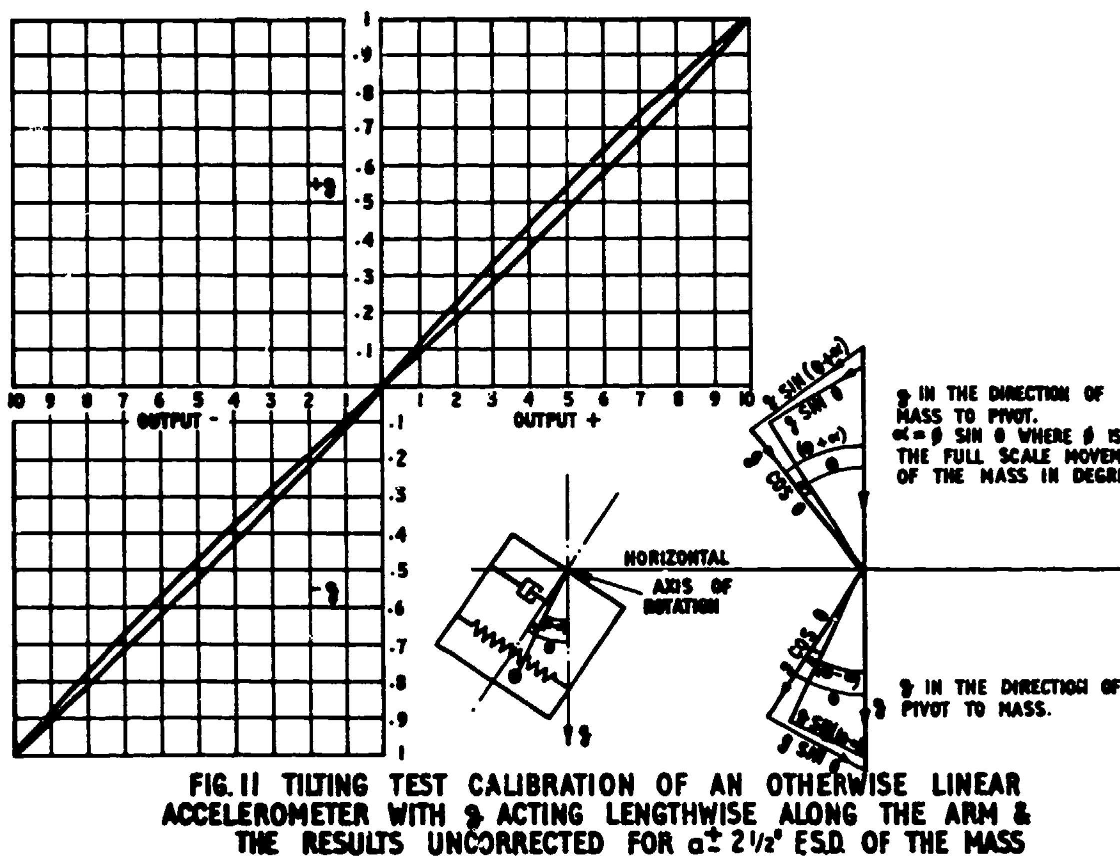


FIG. 11 TILTING TEST CALIBRATION OF AN OTHERWISE LINEAR ACCELEROMETER WITH g ACTING LENGTHWISE ALONG THE ARM & THE RESULTS UNCORRECTED FOR a± 2½° E.S.D. OF THE MASS

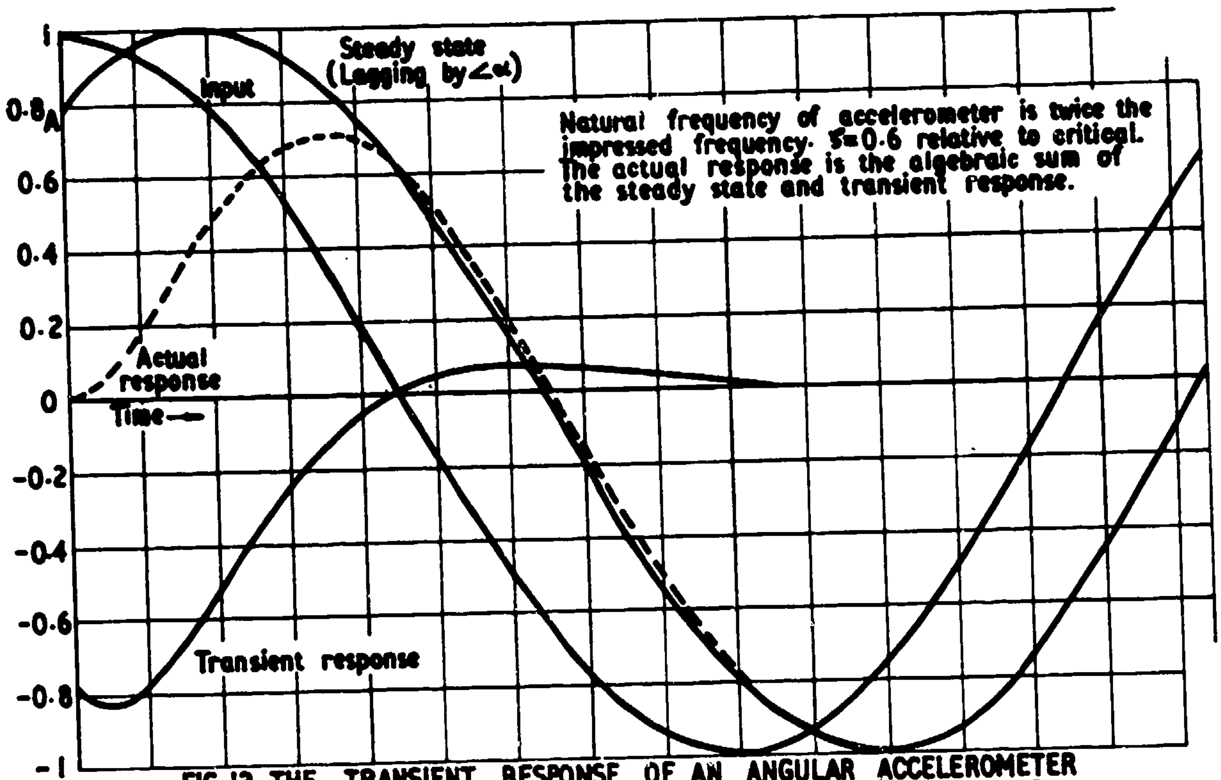


FIG. 12 THE TRANSIENT RESPONSE OF AN ANGULAR ACCELEROMETER WHEN IT IS SUDDENLY SUBJECTED TO A SINUSOIDAL ANGULAR ACCELERATION AT ITS PEAK VALUE.

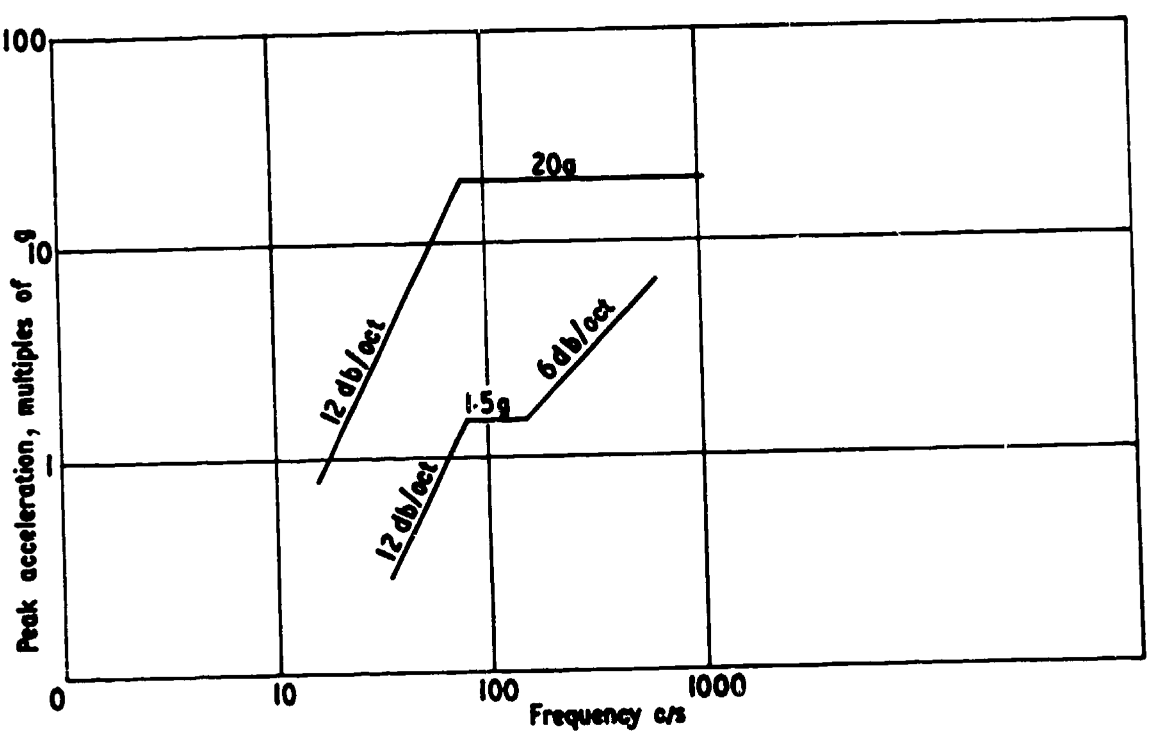
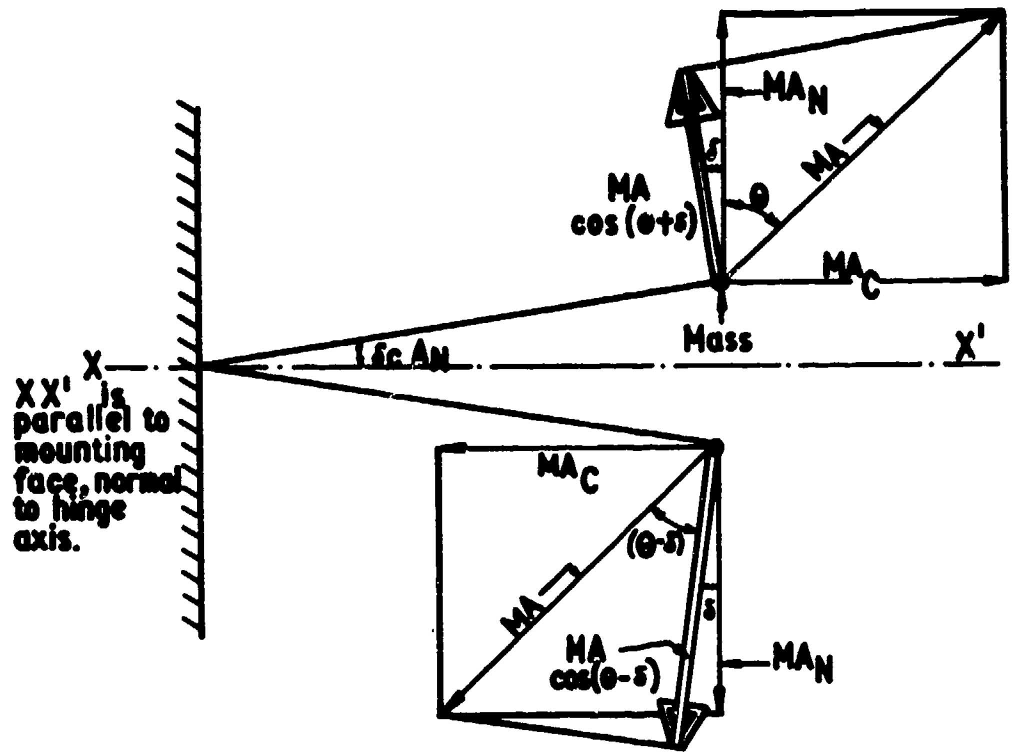


FIG. 13 TYPICAL AIRCRAFT VIBRATION SPECTRA



δ_c = Angle moved through by pick off to produce signal necessary to counteract unit acceleration force

$A = A_0 \sin \omega t$ = vibrational acceleration acting at angle θ to sensitive axis

A_N = Component of vibrational acceleration along sensitive axis

A_C = Component of vibrational acceleration along pendulous arm

FIG. 14 RECTIFICATION EFFECTS ON AN ACCELEROMETER SUBJECTED TO A VIBRATIONAL ACCELERATION ACTING AT ANGLE θ TO THE SENSITIVE AXIS.

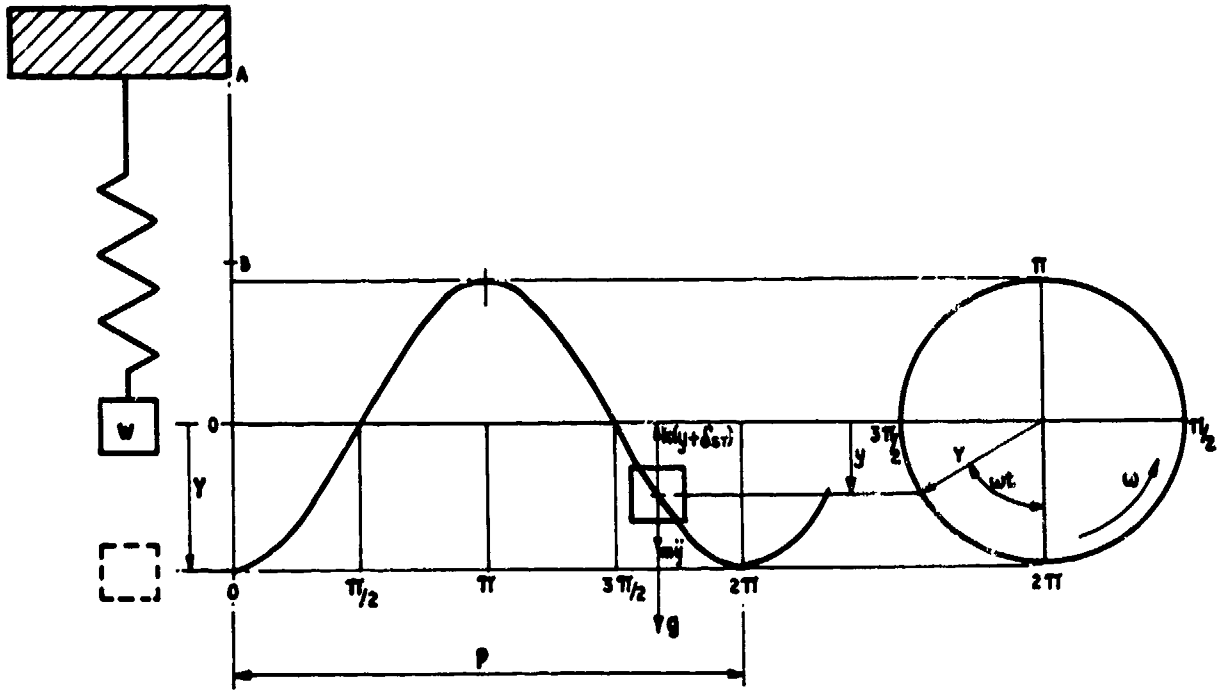


FIG. 15 FREE VIBRATIONS WITHOUT DAMPING

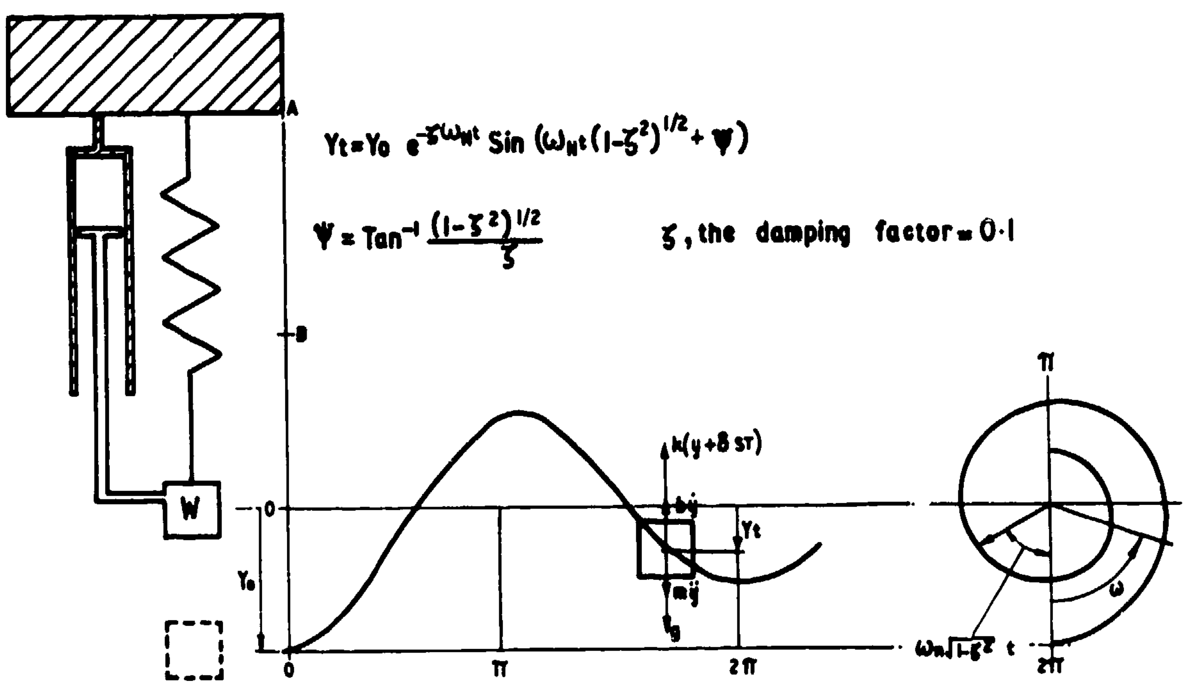


FIG. 16 FREE VIBRATIONS WITH DAMPING

DISTRIBUTION OF UNCLASSIFIED AGARD PUBLICATIONS

NOTE: Initial distributions of AGARD unclassified publications are made to NATO Member Nations through the following National Distribution Centres. Further copies are sometimes available from these Centres, but if not may be purchased in Microfiche or photocopy form from the Purchase Agencies listed below. THE UNITED STATES NATIONAL DISTRIBUTION CENTRE (NASA) DOES NOT HOLD STOCKS OF AGARD PUBLICATIONS, AND APPLICATIONS FOR FURTHER COPIES SHOULD BE MADE DIRECT TO THE APPROPRIATE PURCHASE AGENCY (NTIS).

NATIONAL DISTRIBUTION CENTRES

BELGIUM
Coordonnateur AGARD – VSL
Etat-Major de la Force Aérienne
Caserne Prince Baudouin
Place Dailly, 1030 Bruxelles

CANADA
Defence Scientific Information Service
Department of National Defence
Ottawa, Ontario K1A 0Z3

DENMARK
Danish Defence Research Board
Østerbrogades Kaserne
Copenhagen Ø

FRANCE
O.N.E.R.A. (Direction)
29, Avenue de la Division Leclerc
92, Châtillon sous Bagneux

GERMANY
Zentralstelle für Luftfahrtdokumentation und Information
8 München 86
Postfach 860881

GREECE
Hellenic Armed Forces Command
D Branch, Athens

ICELAND
Director of Aviation
c/o Flugrad
Reykjavik

ITALY
Aeronautica Militare
Ufficio del Delegato Nazionale all'AGARD
3, Piazzale Adenauer
Roma/EUR

LUXEMBOURG
See Belgium

NETHERLANDS
Netherlands Delegation to AGARD
National Aerospace Laboratory, NLR
P.O. Box 126
Delft

NORWAY
Norwegian Defence Research Establishment
Main Library
P.O. Box 25
N-2007 Kjeller

PORTUGAL
Direccao do Servico de Material
da Forca Aerea
Rua de Escola Politecnica 42
Lisboa
Attn: AGARD National Delegate

TURKEY
Turkish General Staff (ARGE)
Ankara

UNITED KINGDOM
Defence Research Information Centre
Station Square House
St. Mary Cray
Orpington, Kent BR5 3RE

UNITED STATES
National Aeronautics and Space Administration (NASA)
Langley Field, Virginia 23365
Attn: Report Distribution and Storage Unit
(See Note above)

PURCHASE AGENCIES

Microfiche or Photocopy
National Technical
Information Service (NTIS)
5285 Port Royal Road
Springfield
Virginia 22151, USA

Microfiche
ESRO/ELDO Space
Documentation Service
European Space
Research Organization
114, Avenue Charles de Gaulle
92200 Neuilly sur Seine, France

Microfiche
Technology Reports
Centre (DTI)
Station Square House
St. Mary Cray
Orpington, Kent BR5 3RF
England

Requests for microfiche or photocopies of AGARD documents should include the AGARD serial number, title, author or editor, and publication date. Requests to NTIS should include the NASA accession report number.

* * *

Full bibliographical references and abstracts of AGARD publications are given in the following bi-monthly abstract journals:

Scientific and Technical Aerospace Reports (STAR),
published by NASA,
Scientific and Technical Information Facility
P.O. Box 33, College Park
Maryland 20740, USA

Government Reports Announcements (GRA),
published by the National Technical
Information Services, Springfield
Virginia 22151, USA

Printed by Technical Editing and Reproduction Ltd
Harford House, 7-9 Charlotte St, London, W1P 1HD